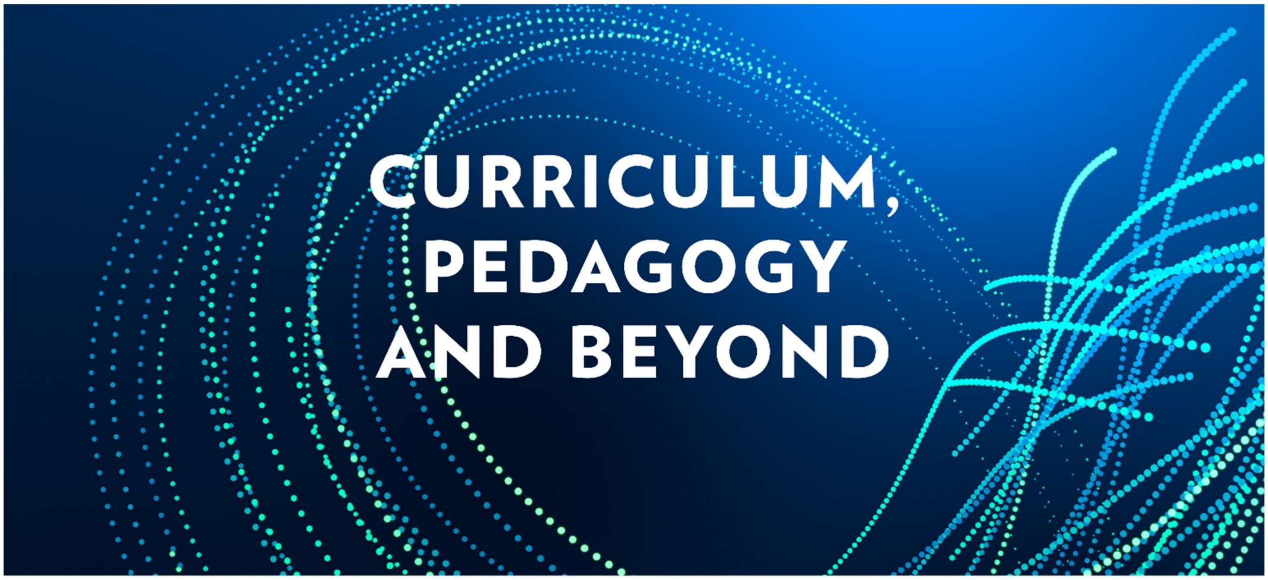

THE MATHEMATICAL ASSOCIATION OF VICTORIA

MAV24
CONFERENCE

NETWORK DECISION TOOLS IN VCE GENERAL MATHS

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Brian Lannen, Murray Mathematics Curriculum Services

Contents

Session A - Scheduling problems and critical path analysis

[2024 NHT General Maths Paper 2 Q 16]

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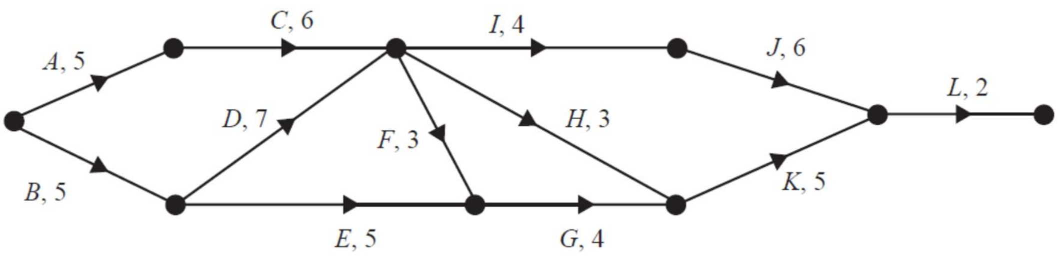
Question 16 (5 marks)

The Tilt-A-Whirl attraction is being upgraded. It will be closed for the duration of the upgrade.

The upgrade involves 12 activities, A to L.

The directed network below shows the activities and their completion times, in days.



a. List the activities that have exactly one immediate predecessor. 1 mark

b. What is the minimum number of days the Tilt-A-Whirl will need to be closed to complete this project? 1 mark

c. What is the latest starting time, in days, for activity I? 1 mark

d. Which activity has the longest float time? 1 mark

The management of the theme park decides that the Tilt-A-Whirl attraction will be closed for too long to complete this project. They give the project manager a budget to reduce the overall completion time.

The project manager is able to hire extra people to reduce the time of some activities, represented in the table below. Each of the activities can be reduced by a maximum of two days.

Activity	Daily cost
A	\$2000
B	\$2000
D	\$2500
E	\$1000
G	\$1500
H	\$1200

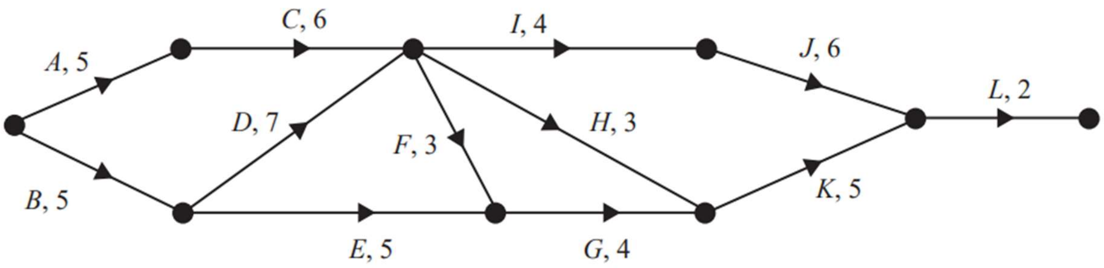
e. Complete the table below, showing the reductions in individual activity times that would achieve the maximum reduction in completion time for the minimum cost. 1 mark

Activity	Reduction in completion time (0, 1 or 2 days)
A	
B	
D	
E	
G	
H	

[2024 NHT General Maths Paper 2 Q 16]

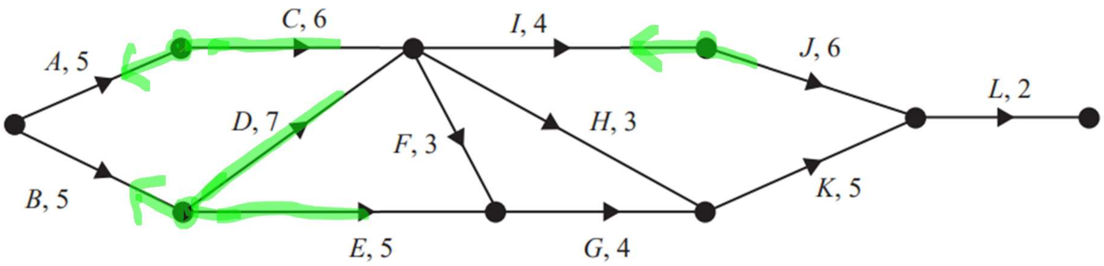
The Tilt-A-Whirl attraction is being upgraded. It will be closed for the duration of the upgrade. The upgrade involves 12 activities *A* to *L*

The directed network below shows the activities and their completion times, in days.



a. List the activities that have exactly one immediate predecessor.

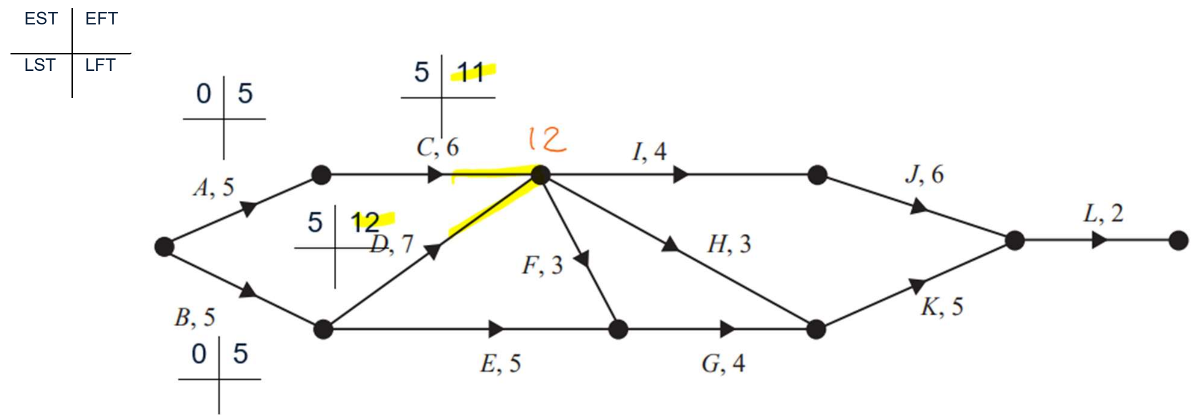
One Immediate Predecessor – On each vertex, go backward and count for **one** edge.

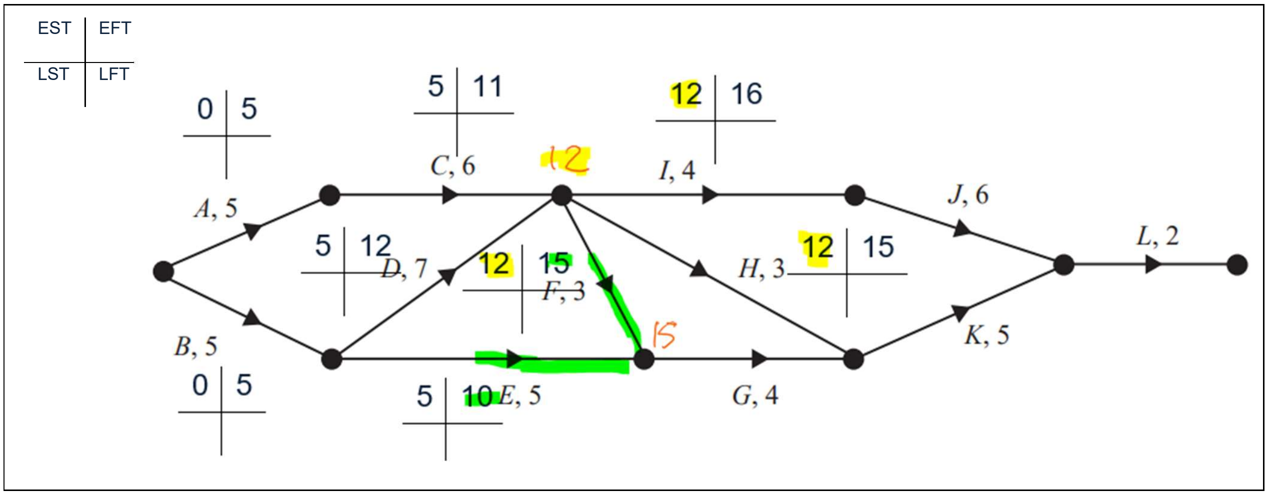


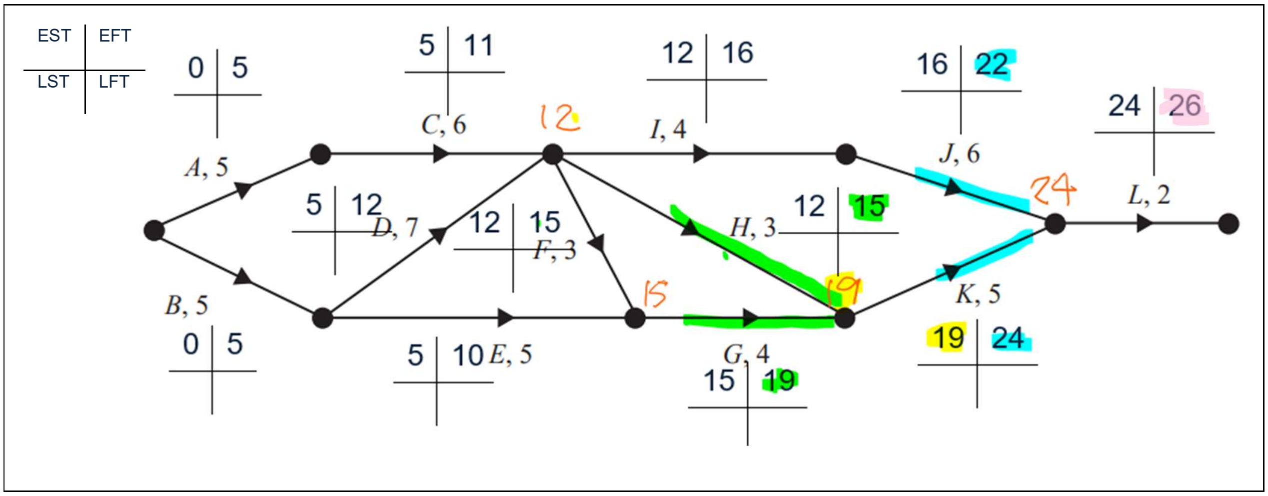
Activity C, D, E and J, each has one immediate predecessor

b. What is the minimum number of days the Tilt-A-Whirl will need to be closed to complete this project?

Earliest Start Time (EST)	Earliest Finishing Time (EFT) = EST + Duration	Forward Scan and choose the biggest
Latest Start Time (LST)	Latest Finishing Time (LFT) = LST + Duration	Backward Scan and choose the smallest

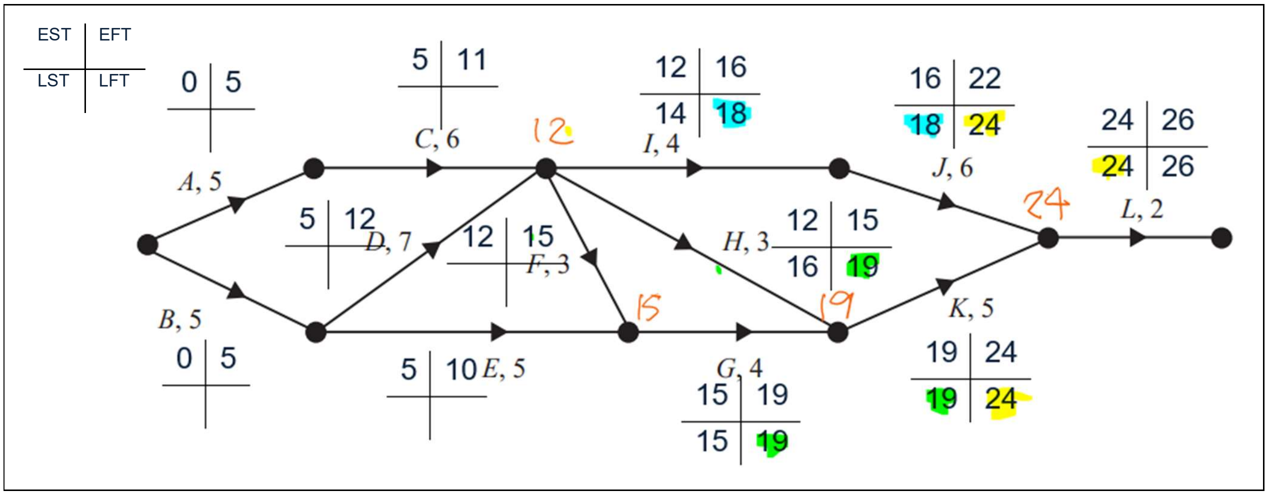




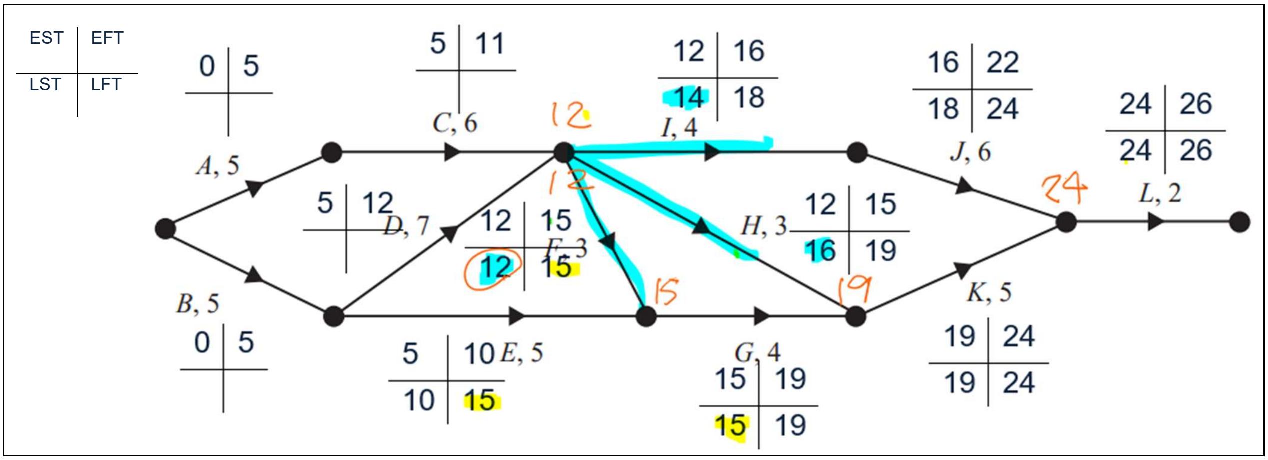


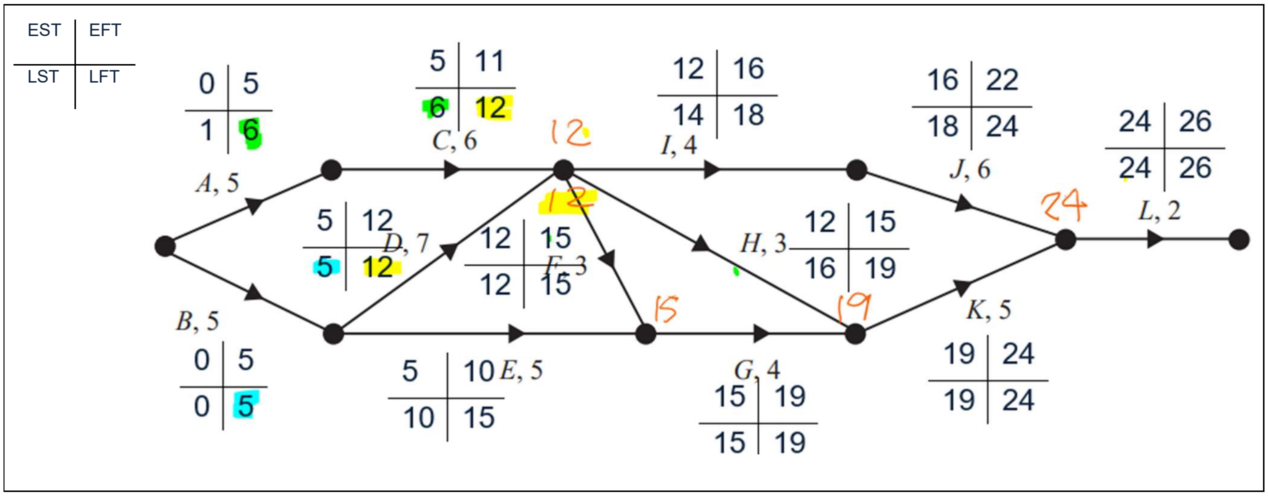
Tilt-A-Whirl needs to be closed for a minimum of **26 days**

c. What is the latest starting time, in days, for activity *I*?

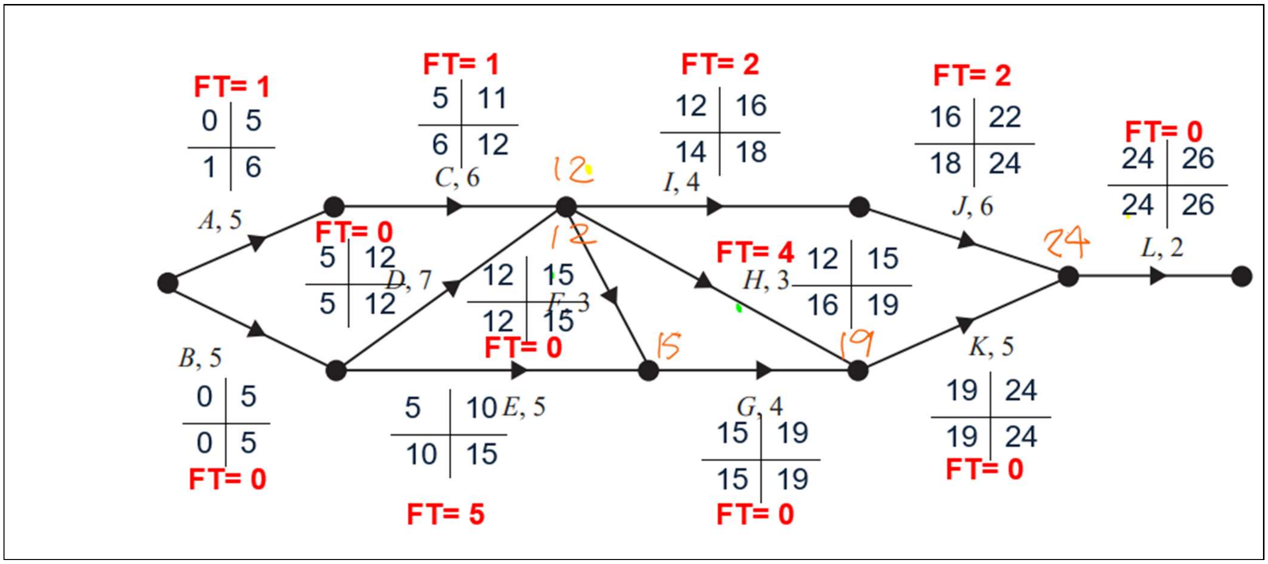


Latest start time for activity *I* is Day 14.





d. Which activities has the longest float time?
Float Time = LFT – EFT = (LST + duration) – (EST + duration) = LST – EST



Activity E has the longest Float Time (5 days)

Additional remark: Activities with FT = 0 form the critical path: B→D→F→G→K→L

We could also set up a spreadsheet on CAS for Project Management analysis

	A activity	**B** predecessor	**C** duration	**D** EST	**E** EFT	**F** successor	**G** LST	**H** LFT	**I** FloatTime
=									
1									
2									

Step One: Copy information from predecessor table to CAS table

	A activity	**B** predecessor	**C** duration	**D** EST	**E** EFT	**F** successor	**G** LST	**H** LFT	**I** FloatTime
=									
1	A	No	5						
2	B	No	5						
3	C	A	6						
4	D	B	7						
5	E	B	5						
6	F	C, D	3						
7	G	E, F	4						
8	H	C, D	3						
9	I	C, D	4						
10	J	I	6						
11	K	G,H	5						
12	L	J, K	2						

CAS Useful short cut key

Ctrl 7 – goto top of list

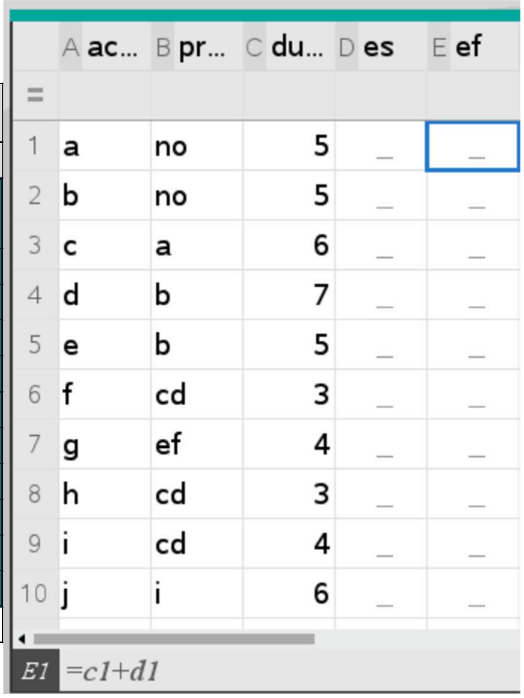
Crtl 1 – goto bottom of list

Step Two: EFT = EST + Duration

In Cell E1, write =b1+c1 and then fill down

	A activity	**B** predecessor	**C** duration	**D** EST	**E** EFT
=					
1	A	No	5		**=c1+d1**
2	B	No	5		
3	C	A	6		
4	D	B	7		
5	E	B	5		
6	F	C, D	3		
7	G	E, F	4		
8	H	C, D	3		
9	I	C, D	4		
10	J	I	6		
11	K	G,H	5		
12	L	J, K	2		

CAS view:

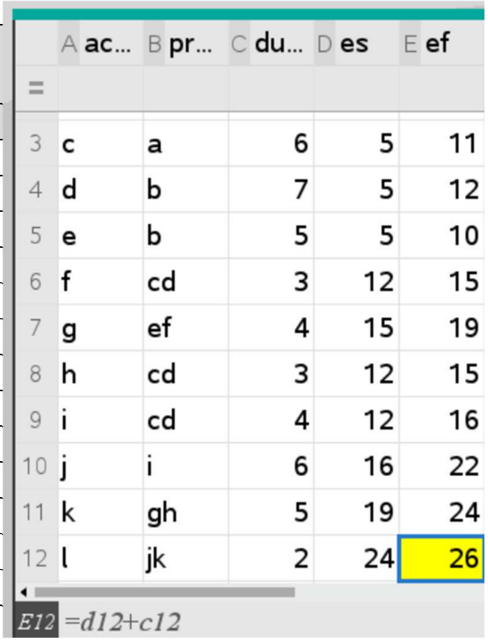


	A ac…	B pr…	C du…	D es	E ef
=					
1	a	no	5	_	_
2	b	no	5	_	_
3	c	a	6	_	_
4	d	b	7	_	_
5	e	b	5	_	_
6	f	cd	3	_	_
7	g	ef	4	_	_
8	h	cd	3	_	_
9	i	cd	4	_	_
10	j	i	6	_	_

E1 =c1+d1

Step Three: Fill in EST = EFT of predecessor. Use Max( ) for more than one predecessor.

	A activity	**B** predecessor	**C** duration	**D** EST	**E** EFT
=					
1	A	No	5	**0**	**This should be automatically filled as per formula written before**
2	B	No	5	**0**	
3	C	A	6	**=e1**	
4	D	B	7	**=e2**	
5	E	B	5	**=e2**	
6	F	C, D	3	**=max(e3,e4)**	
7	G	E, F	4	**= max(e5,e6)**	
8	H	C, D	3	**= max(e3,e4)**	
9	I	C, D	4	**= max(e3,e4)**	
10	J	I	6	**=e9**	
11	K	G,H	5	**=max(e7,e8)**	
12	L	J, K	2	**=max(e10,e11)**	
13					



	A ac...	B pr...	C du...	D es	E ef
=					
3	c	a	6	5	11
4	d	b	7	5	12
5	e	b	5	5	10
6	f	cd	3	12	15
7	g	ef	4	15	19
8	h	cd	3	12	15
9	i	cd	4	12	16
10	j	i	6	16	22
11	k	gh	5	19	24
12	l	jk	2	24	26

E12 =d12+c12

Result after Step Three.

1. From here we know that completion time is EFT
2. LFT is the same as completion time

	A activity	**B** predecessor	**C** duration	**D** EST	**E** EFT	**F** successor	**G** LST	**H** LFT	**I** FloatTime
=									
1	A	No	5	0	**5**				
2	B	No	5	0	**5**				
3	C	A	6	5	**11**				
4	D	B	7	5	**12**				
5	E	B	5	5	**10**				
6	F	C, D	3	12	**15**				
7	G	E, F	4	15	**19**				
8	H	C, D	3	12	**15**				
9	I	C, D	4	12	**16**				
10	J	I	6	16	**22**				
11	K	G,H	5	19	**24**				
12	L	J, K	2	24	**26**			**=e12**	
13									

Continue next page

Step Four: Fill in information of successor (Read from **table** or **network**)

	A activity	**B** predecessor	**C** duration	**D** EST	**E** EFT	**F** successor	**G** LST	**H** LFT	**I** FloatTime
=									
1	A	No	5	0	**5**	**C**			
2	B	No	5	0	**5**	**D, E**			
3	C	A	6	5	**11**	**F, H, I**			
4	D	B	7	5	**12**	**F, H, I**			
5	E	B	5	5	**10**	**G**			
6	F	C, D	3	12	**15**	**G**			
7	G	E, F	4	15	**19**	**K**			
8	H	C, D	3	12	**15**	**K**			
9	I	C, D	4	12	**16**	**J**			
10	J	I	6	16	**22**	**L**			
11	K	G,H	5	19	**24**	**L**			
12	L	J, K	2	24	**26**	**No**		**26**	
13									

Step Five: LST of each activity = LFT - duration

	A activity	**B** predecessor	**C** duration	**D** EST	**E** EFT	**F** successor	**G** LST	**H** LFT	**I** FloatTime
=									
1	A	No	5	0	5	C	**=h1-c1**		
2	B	No	5	0	5	D, E			
3	C	A	6	5	11	F, H, I			
4	D	B	7	5	12	F, H, I			
5	E	B	5	5	10	G			
6	F	C, D	3	12	15	G			
7	G	E, F	4	15	19	K			
8	H	C, D	3	12	15	K			
9	I	C, D	4	12	16	J			
10	J	I	6	16	22	L			
11	K	G,H	5	19	24	L			
12	L	J, K	2	24	26	No	▼	**24**	
13									

Continue next page

Step Six: LFT of each activity = LST of successor activity. Use min( ) for more than one activity.

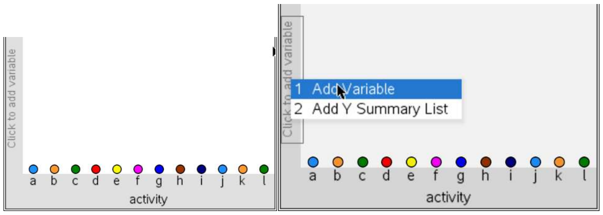
	A activity	**B** predecessor	**C** duration	**D** EST	**E** EFT	**F** successor	**G** LST	**H** LFT	**I** FT
=									
1	A	No	5	0	5	C	This should be automatically filled as per formula written before	**=g3**	
2	B	No	5	0	5	D, E		**=min(g4,g5)**	
3	C	A	6	5	11	F, H, I		**=min(min(g6,g8),g9)**	
4	D	B	7	5	12	F, H, I		**=min(min(g6,g8),g9)**	
5	E	B	5	5	10	G		**=g7**	
6	F	C, D	3	12	15	G		**=g7**	
7	G	E, F	4	15	19	K		**=g11**	
8	H	C, D	3	12	15	K		**=g11**	
9	I	C, D	4	12	16	J		**=g10**	
10	J	I	6	16	22	L		**=g12**	
11	K	G,H	5	19	24	L		**=g12**	
12	L	J, K	2	24	26	No		**26**	
13									

Step Seven: Generate Float Time of each activity, Float Time = LFT – EFT*

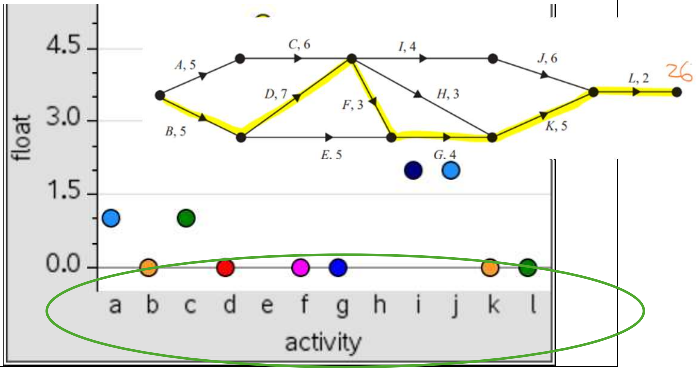
***FT can be calculated as FT = LST – EST**

	A activity	**B** predecessor	**C** duration	**D** EST	**E** EFT	**F** successor	**G** LST	**H** LFT	**I** FloatTime
=									**=LFT-EFT**
1	A	No	5	0	5	C	**10**	**6**	**1**
2	B	No	5	0	5	D, E	**0**	**5**	**0**
3	C	A	6	5	11	F, H, I	**6**	**12**	**1**
4	D	B	7	5	12	F, H, I	**5**	**12**	**0**
5	E	B	5	5	10	G	**10**	**15**	**5**
6	F	C, D	3	12	15	G	**12**	**15**	**0**
7	G	E, F	4	15	19	K	**15**	**19**	**0**
8	H	C, D	3	12	15	K	**16**	**19**	**4**
9	I	C, D	4	12	16	J	**14**	**18**	**2**
10	J	I	6	16	22	L	**18**	**24**	**2**
11	K	G,H	5	19	24	L	**19**	**24**	**0**
12	L	J, K	2	24	26	No	**24**	**26**	**0**
13									

Plot 'Data and Statistics' page to see Critical activities.
Choose activity on x-axis
Mouse over y-axis; Ctrl menu→ Add Y var
Choose Float Time



Float time = 0
B – D – F – G – K - L



The management of the theme park decides that the Tilt-A-Whirl attraction will be closed for too long to complete this project. They give the project manager a budget to reduce the overall completion time. The project manager is able to hire extra people to reduce the time of some activities, represented in the table below. **Each of the activities can be reduced by a maximum of two days.**

Activity	Daily cost
A	$2000
B	$2000
D	$2500
E	$1000
G	$1500
H	$1200

To Consider Crashing, Start with Critical Path activities. Notice that Critical path changes every time as an activity is crashed. CAS Data View might be handy to notice new critical path.

At this stage, Critical path is B D F G K L
ON it, Activity B, D and G can be crashed. Activity G is the cheapest to crash.

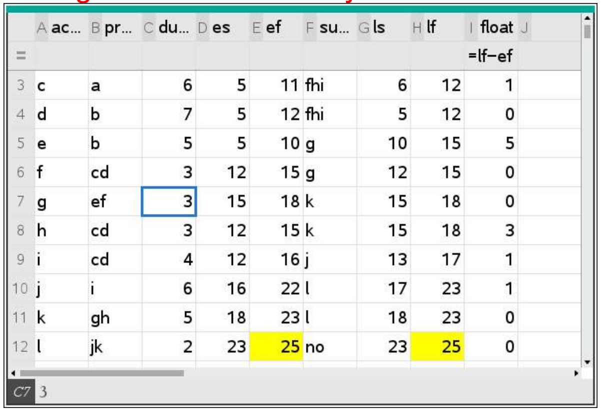
This table lists all paths from start to finish

1.	A (5)	C (6)	I (4)	J (6)	L (2)		5+6+4+6+2 = 23
2.	A (5)	C (6)	H (3)	K (5)	L (2)		5+6+3+5+2= 21
3.	A (5)	C (6)	F (3)	G (4)	K (5)	L (2)	5+6+3+4+5+2 = 25
4.	B (5)	D (7)	I (4)	J (6)	L (2)		5+7+4+6+2 = 24
5.	B (5)	E (5)	G (4)	K (5)	L (2)		5+5+4+5+2 = 21
6.	B (5)	D (7)	F (3)	G (4)	K (5)	L (2)	5+7+3+4+5+2= 26
7.	B (5)	D (7)	H (3)	K (5)	L (2)		5+7+3+5+2 = 22

Crash G for 1 day

1.	A (5)	C (6)	I (4)	J (6)	L (2)		23
2.	A (5)	C (6)	H (3)	K (5)	L (2)		21
3.	A (5)	C (6)	F (3)	G (3)	K (5)	L (2)	24
4.	B (5)	D (7)	I (4)	J (6)	L (2)		24
5.	B (5)	E (5)	G (3)	K (5)	L (2)		20
6.	B (5)	D (7)	F (3)	G (3)	K (5)	L (2)	25
7.	B (5)	D (7)	H (3)	K (5)	L (2)		22

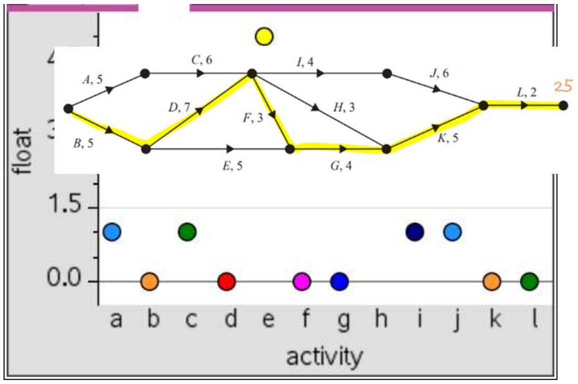
Change duration of activity G to 3



	A ac…	B pr…	C du…	D es	E ef	F su…	G ls	H lf	I float	J
=									=lf−ef	
3	c	a	6	5	11	fhi	6	12	1	
4	d	b	7	5	12	fhi	5	12	0	
5	e	b	5	5	10	g	10	15	5	
6	f	cd	3	12	15	g	12	15	0	
7	g	ef	3	15	18	k	15	18	0	
8	h	cd	3	12	15	k	15	18	3	
9	i	cd	4	12	16	j	13	17	1	
10	j	i	6	16	22	l	17	23	1	
11	k	gh	5	18	23	l	18	23	0	
12	l	jk	2	23	25	no	23	25	0	

C7 3

Check Critical Path:
B D F G K L



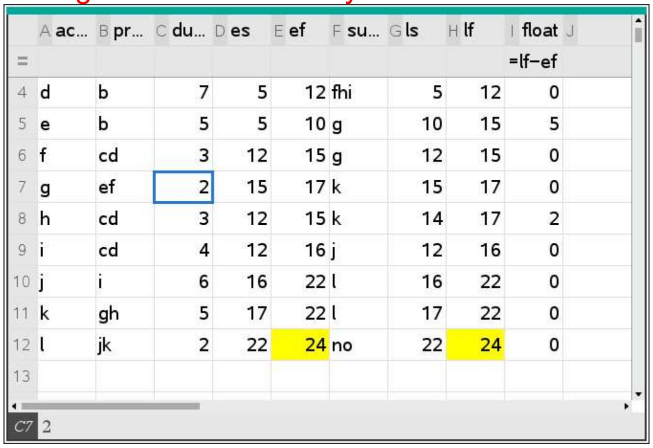
Notice new **minimum completion time becomes 25 days**.

Critical path is still the same B D F G K L, ie G is still the cheapest to crash out of B, D and G.

Crash G for another 1 day (total 2 days)

1.	A (5)	C (6)	I (4)	J (6)	L (2)		23
2.	A (5)	C (6)	H (3)	K (5)	L (2)		21
3.	A (5)	C (6)	F (3)	G (2)	K (5)	L (2)	23
4.	B (5)	D (7)	I (4)	J (6)	L (2)		24
5.	B (5)	E (5)	G (2)	K (5)	L (2)		19
6.	B (5)	D (7)	F (3)	G (2)	K (5)	L (2)	24
7.	B (5)	D (7)	H (3)	K (5)	L (2)		22

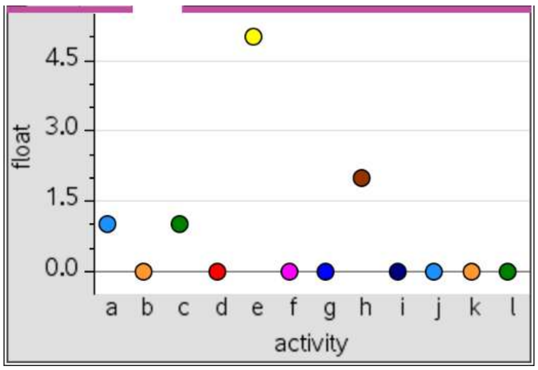
Change duration of activity G to **2**



	A ac...	B pr...	C du...	D es	E ef	F su...	G ls	H lf	I float	J
=									=lf−ef	
4	d	b	7	5	12	fhi	5	12	0	
5	e	b	5	5	10	g	10	15	5	
6	f	cd	3	12	15	g	12	15	0	
7	g	ef	2	15	17	k	15	17	0	
8	h	cd	3	12	15	k	14	17	2	
9	i	cd	4	12	16	j	12	16	0	
10	j	i	6	16	22	l	16	22	0	
11	k	gh	5	17	22	l	17	22	0	
12	l	jk	2	22	24	no	22	24	0	
13										

C7 2

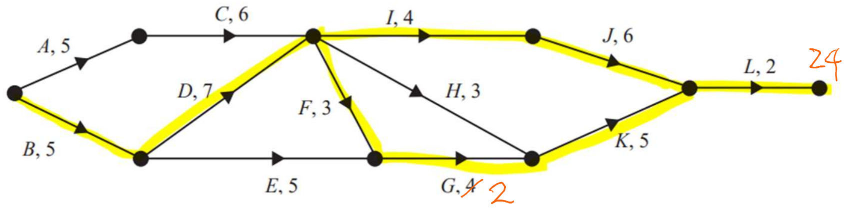
Check Critical Path:



Notice new critical paths form

Notice new **minimum completion time becomes 24 days** and new Critical paths are formed

B D I J L and B D F G K L



Activity	Daily cost
A	$2000
B	$2000
D	$2500
E	$1000
~~G~~	$1500 x2
H	$1200

We have now exhausted activity G and used $3000.

We now have two critical paths:
B D I J L and B D F G K L

G is grey out as we can't reduce its duration any more

On both critical paths, only B and D can be crashed and B is cheapest

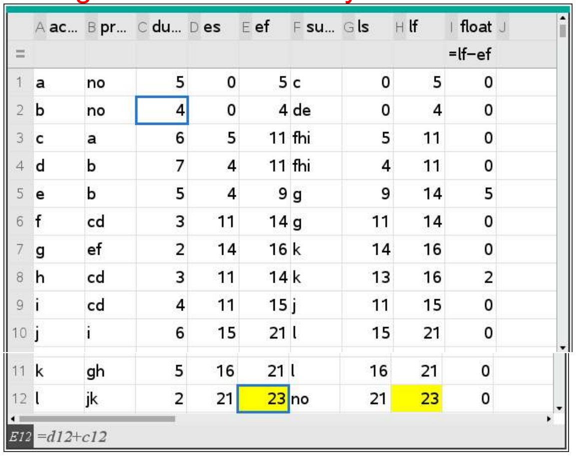
Activity	Daily cost
A	$2000
B	$2000
D	$2500
E	$1000
G	$1500
H	$1200

1.	A (5)	C (6)	I (4)	J (6)	L (2)		23
2.	A (5)	C (6)	H (3)	K (5)	L (2)		21
3.	A (5)	C (6)	F (3)	G (2)	K (5)	L (2)	23
4.	B (5)	D (7)	I (4)	J (6)	L (2)		24
5.	B (5)	E (5)	G (2)	K (5)	L (2)		19
6.	B (5)	D (7)	F (3)	G (2)	K (5)	L (2)	24
7.	B (5)	D (7)	H (3)	K (5)	L (2)		22

Crash B for 1 day.

1.	A (5)	C (6)	I (4)	J (6)	L (2)		23
2.	A (5)	C (6)	H (3)	K (5)	L (2)		21
3.	A (5)	C (6)	F (3)	G (2)	K (5)	L (2)	23
4.	B (4)	D (7)	I (4)	J (6)	L (2)		23
5.	B (4)	E (5)	G (2)	K (5)	L (2)		18
6.	B (4)	D (7)	F (3)	G (2)	K (5)	L (2)	23
7.	B (4)	D (7)	H (3)	K (5)	L (2)		21

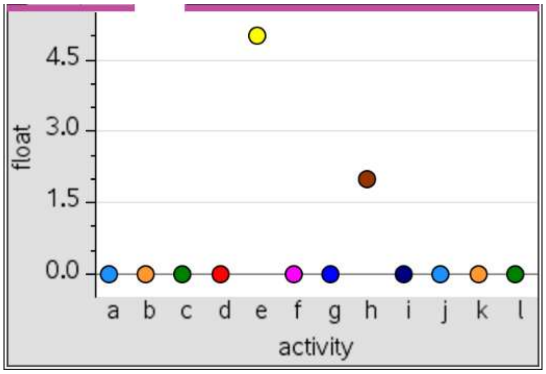
Change duration of activity B to 4



	A ac...	B pr...	C du...	D es	E ef	F su...	G ls	H lf	I float	J
=									=lf−ef	
1	a	no	5	0	5	c	0	5	0	
2	b	no	4	0	4	de	0	4	0	
3	c	a	6	5	11	fhi	5	11	0	
4	d	b	7	4	11	fhi	4	11	0	
5	e	b	5	4	9	g	9	14	5	
6	f	cd	3	11	14	g	11	14	0	
7	g	ef	2	14	16	k	14	16	0	
8	h	cd	3	11	14	k	13	16	2	
9	i	cd	4	11	15	j	11	15	0	
10	j	i	6	15	21	l	15	21	0	
11	k	gh	5	16	21	l	16	21	0	
12	l	jk	2	21	23	no	21	23	0	

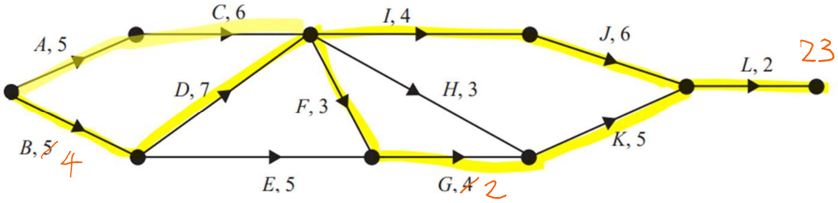
E12 =d12+c12

Check Critical Path:



Notice new **minimum completion time becomes 23 days** and new Critical paths are formed

FOUR Critical Paths



Activity	Daily cost
A	$2000
B	$2000 x 1
D	$2500
E	$1000
~~*G*~~	$1500 x 2
H	$1200

We have now crashed activity G for 2 days ($3000) and activity B for 1 day ($2000).

We now have **FOUR** critical paths:

G is grey out as we can't reduce its duration any more

On these critical paths,

A (cheapest), B (cheapest), and D can be crashed.

Activity	Daily cost
A	$2000
B	$2000 ×1
D	$2500
E	$1000
~~G~~	$1500 ×2
H	$1200

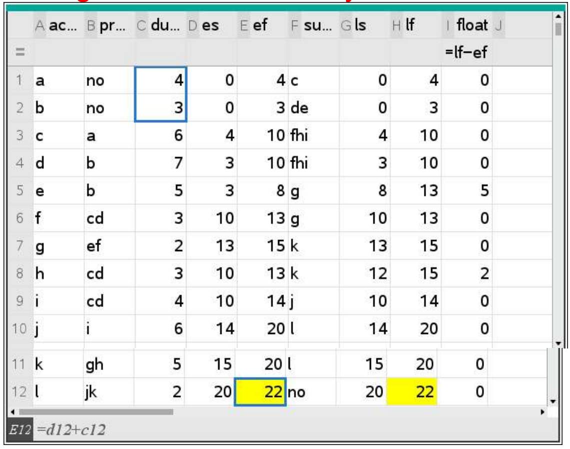
1.	A (5)	C (6)	I (4)	J (6)	L (2)		23
2.	A (5)	C (6)	H (3)	K (5)	L (2)		21
3.	A (5)	C (6)	F (3)	G (2)	K (5)	L (2)	23
4.	B (4)	D (7)	I (4)	J (6)	L (2)		23
5.	B (4)	E (5)	G (2)	K (5)	L (2)		18
6.	B (4)	D (7)	F (3)	G (2)	K (5)	L (2)	23
7.	B (4)	D (7)	H (3)	K (5)	L (2)		21

Not only both A and B are cheapest. We also noticed that if we just crash A, path 1 and 3 reduce to 22 days but not path 4 and 6, this is not going to help to reduce overall completion time. With the same reason, we can't just crash B. Hence, we need to crash both A and B for 1 day each.

Crash B for another 1 day (total 2 days) and crash A for 1 day.

1.	A (4)	C (6)	I (4)	J (6)	L (2)		22
2.	A (4)	C (6)	H (3)	K (5)	L (2)		20
3.	A (4)	C (6)	F (3)	G (2)	K (5)	L (2)	22
4.	B (3)	D (7)	I (4)	J (6)	L (2)		22
5.	B (3)	E (5)	G (2)	K (5)	L (2)		17
6.	B (3)	D (7)	F (3)	G (2)	K (5)	L (2)	22
7.	B (3)	D (7)	H (3)	K (5)	L (2)		20

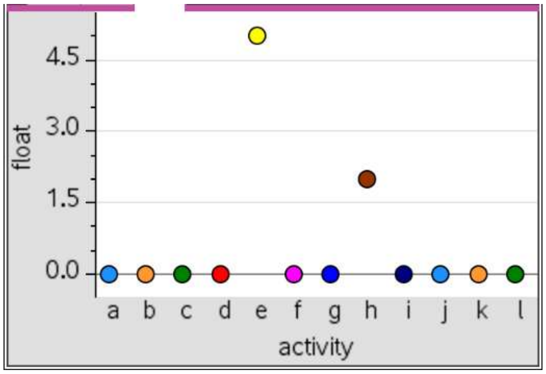
Change duration of activity A to 4 and B to 3



	A ac...	B pr...	C du...	D es	E ef	F su...	G ls	H lf	I float	J
=									=lf−ef	
1	a	no	4	0	4	c	0	4	0	
2	b	no	3	0	3	de	0	3	0	
3	c	a	6	4	10	fhi	4	10	0	
4	d	b	7	3	10	fhi	3	10	0	
5	e	b	5	3	8	g	8	13	5	
6	f	cd	3	10	13	g	10	13	0	
7	g	ef	2	13	15	k	13	15	0	
8	h	cd	3	10	13	k	12	15	2	
9	i	cd	4	10	14	j	10	14	0	
10	j	i	6	14	20	l	14	20	0	
11	k	gh	5	15	20	l	15	20	0	
12	l	jk	2	20	22	no	20	22	0	

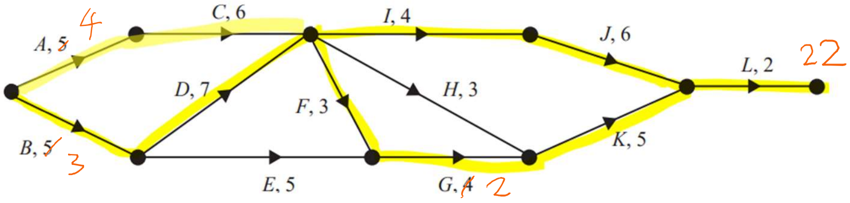
E12 =d12+c12

Check Critical Path:



Notice new **minimum completion time becomes 22 days** and no new Critical paths are formed.

FOUR Critical Paths



Activity	Daily cost
A	$2000 ×1
~~B~~	$2000 ×2
D	$2500
E	$1000
~~G~~	$1500 ×2
H	$1200

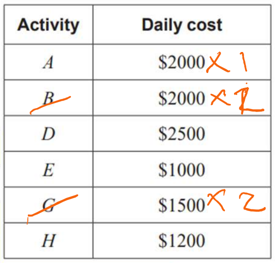
We have now crashed activity G for 2 days ($3000), activity B for 2 day2 ($4000) and activity A for 1 day ($2000)

We now have the same **FOUR** critical paths:

B, G is grey out as we can't reduce its duration any more

On these critical paths,

A (cheapest) and D can be crashed.



Activity	Daily cost
A	$2000 × 1
~~B~~	$2000 × 2
D	$2500
E	$1000
~~G~~	$1500 × 2
H	$1200

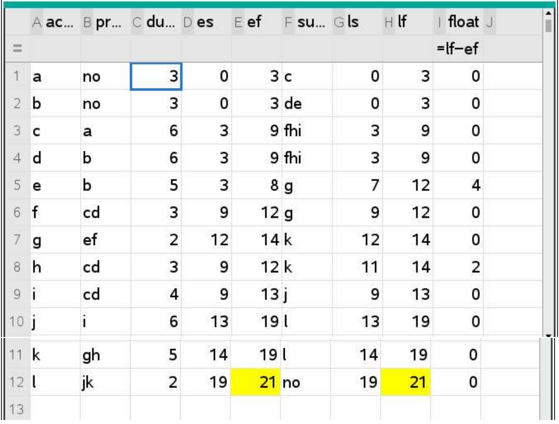
1.	A (4)	C (6)	I (4)	J (6)	L (2)		22
2.	A (4)	C (6)	H (3)	K (5)	L (2)		20
3.	A (4)	C (6)	F (3)	G (2)	K (5)	L (2)	22
4.	B (3)	D (7)	I (4)	J (6)	L (2)		22
5.	B (3)	E (5)	G (2)	K (5)	L (2)		17
6.	B (3)	D (7)	F (3)	G (2)	K (5)	L (2)	22
7.	B (3)	D (7)	H (3)	K (5)	L (2)		20

Although activity A is cheapest, we can't just crash A as A is on only two of the four critical paths. Hence crash A and D, each for 1 day

Crash A for another 1 day (total 2 days) and crash D for 1 day.

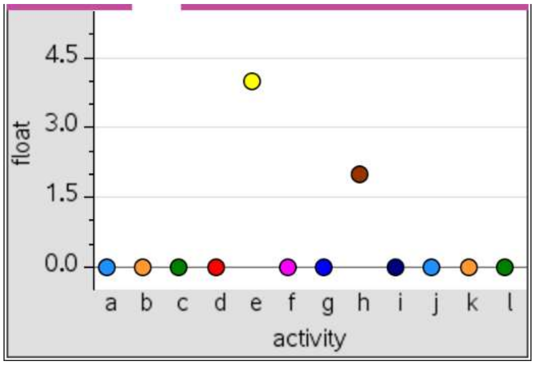
1.	A (3)	C (6)	I (4)	J (6)	L (2)		21
2.	A (3)	C (6)	H (3)	K (5)	L (2)		19
3.	A (3)	C (6)	F (3)	G (2)	K (5)	L (2)	21
4.	B (3)	D (6)	I (4)	J (6)	L (2)		21
5.	B (3)	E (5)	G (2)	K (5)	L (2)		17
6.	B (3)	D (6)	F (3)	G (2)	K (5)	L (2)	21
7.	B (3)	D (6)	H (3)	K (5)	L (2)		19

Change duration of activity A to 4 and B to 3



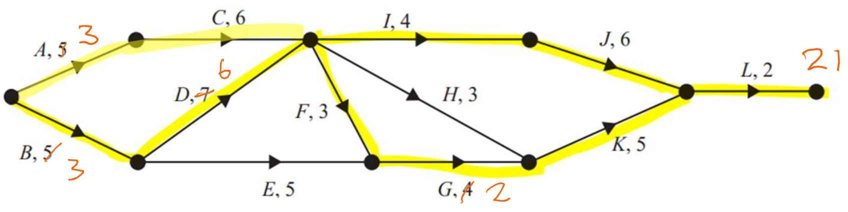
	A ac...	B pr...	C du...	D es	E ef	F su...	G ls	H lf	I float	J
=									=lf−ef	
1	a	no	3	0	3	c	0	3	0	
2	b	no	3	0	3	de	0	3	0	
3	c	a	6	3	9	fhi	3	9	0	
4	d	b	6	3	9	fhi	3	9	0	
5	e	b	5	3	8	g	7	12	4	
6	f	cd	3	9	12	g	9	12	0	
7	g	ef	2	12	14	k	12	14	0	
8	h	cd	3	9	12	k	11	14	2	
9	i	cd	4	9	13	j	9	13	0	
10	j	i	6	13	19	l	13	19	0	
11	k	gh	5	14	19	l	14	19	0	
12	l	jk	2	19	21	no	19	21	0	
13										

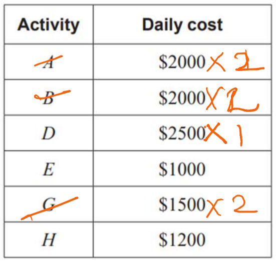
Check Critical Path:



Notice new **minimum completion time becomes 21 days** and no new Critical paths are formed.

FOUR Critical Paths





Activity	Daily cost
~~A~~	$2000 × 2
~~B~~	$2000 × 2
D	$2500 × 1
E	$1000
~~G~~	$1500 × 2
H	$1200

We have now crashed activity G for 2 days ($3000), activity B for 2 day2 ($4000), activity A for 2 days ($4000) and Activity D for 1 day ($2500). **So the answer is A – 2; B – 2; D – 1; E – 0; G – 2; H – 0.**

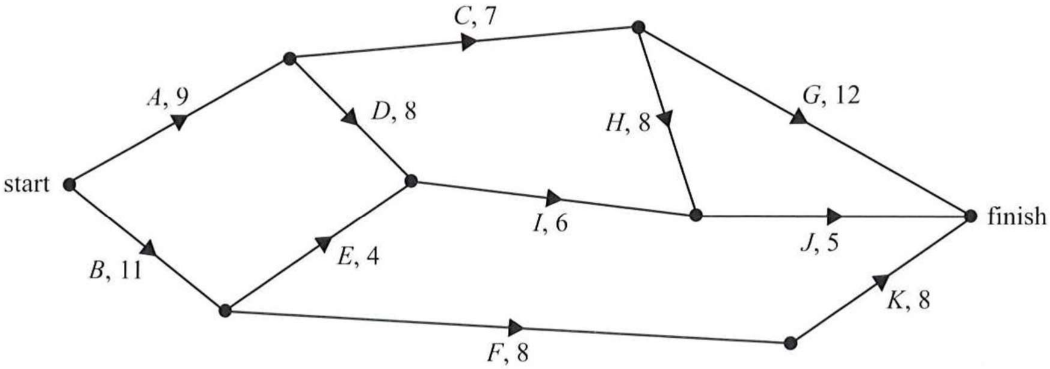
[2024 VCAA General Maths Paper 2 Q15]

Question 15 (5 marks)

An upgrade to the supermarket requires the completion of 11 activities, *A* to *K*.

The directed network below shows these activities and their completion time, in weeks.

The minimum completion time for the project is 29 weeks.



a. Write down the critical path. 1 mark

b. Which activity can be delayed for the longest time without affecting the minimum completion time of the project? 1 mark

Use the following information to answer parts c–e.

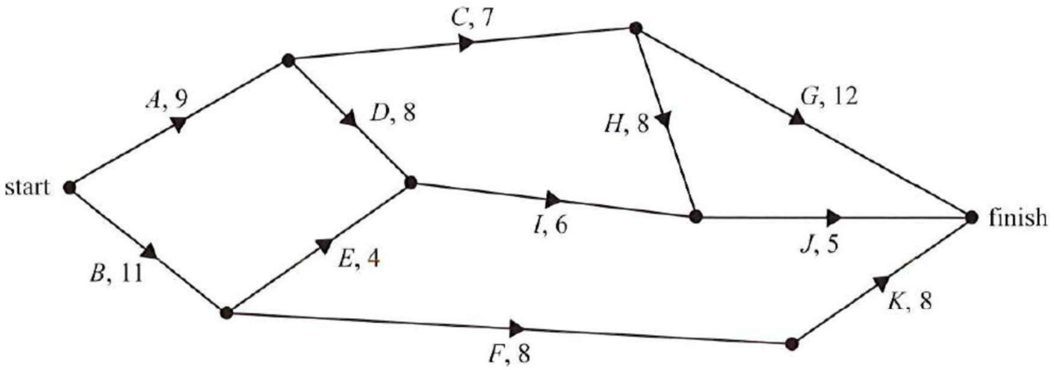
A change is made to the order of activities.

The table below shows the activities and their new latest starting times in weeks.

Activity	**Latest starting time** (weeks)
A	0
B	2
C	10
D	9
E	13
F	14
G	18
H	17
I	19
J	25
K	22

A dummy activity is now required in the network.

c. On the directed network below, draw a directed edge to represent the dummy activity. Include a label. 1 mark



d. What is the new minimum completion time of the project? 1 mark

e. The owners of the supermarket want the project completed earlier.

They will pay to reduce the time of some of the activities.

A reduction in completion time of an activity will incur an additional cost of \$10 000 per week.

Activities can be reduced by a maximum of two weeks.

The minimum number of weeks an activity can be reduced to is seven weeks.

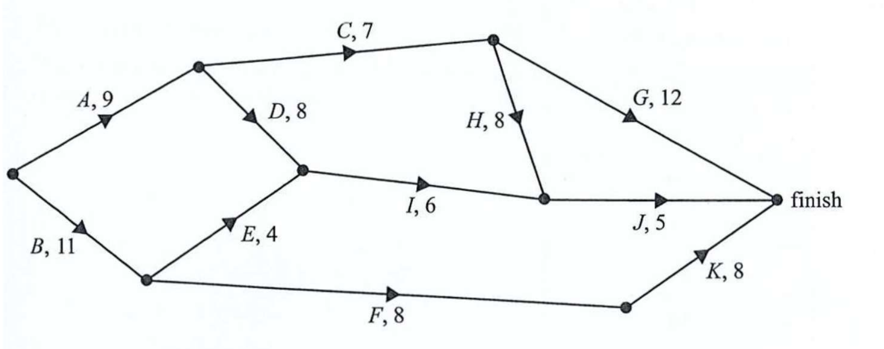
What is the minimum amount the owners of the supermarket will have to pay to reduce the completion time of the project as much as possible? 1 mark

[2024 VCAA General Maths Paper 2 Q15] – CAS spreadsheet method

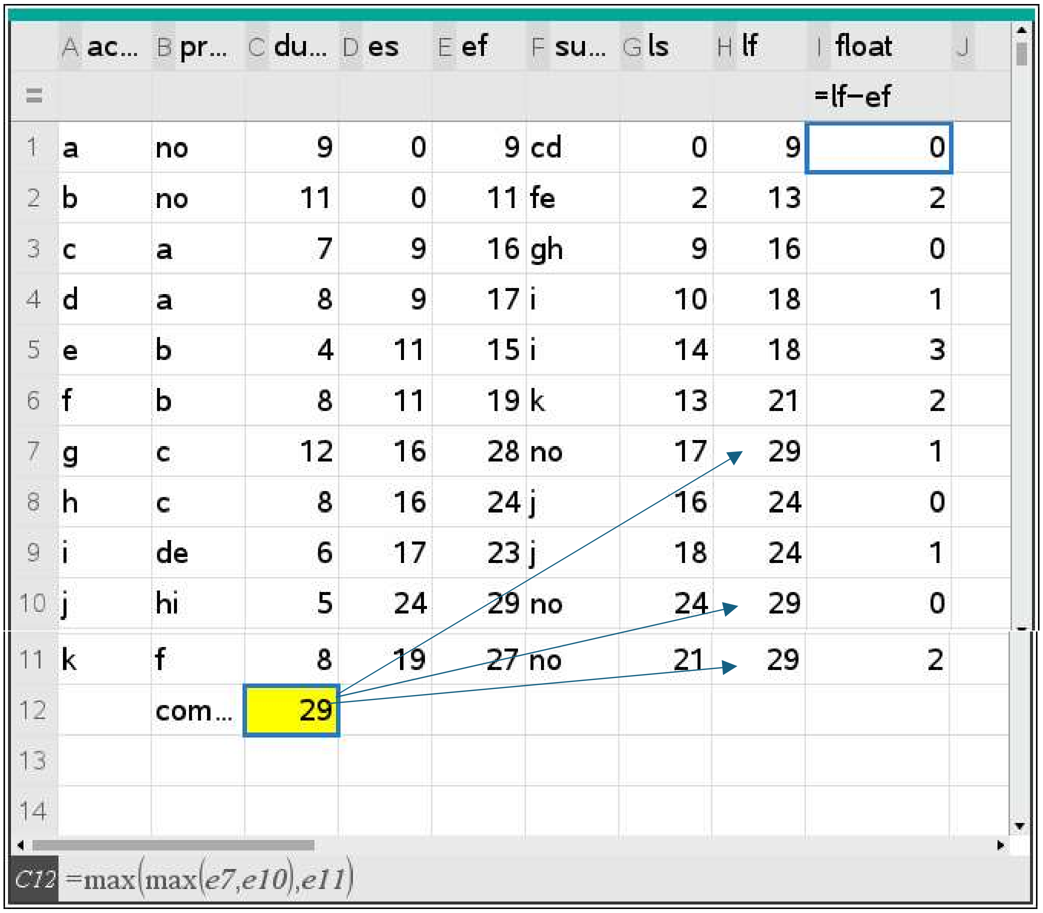
An upgrade to the supermarket requires the completion of 11 activities, *A* to *K*.

The directed network below shows these activities and their completion time, in weeks.

The minimum completion time for the project is 29 weeks.

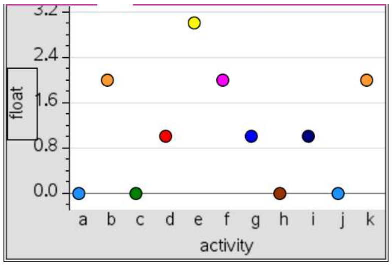


a. Write down the critical path.



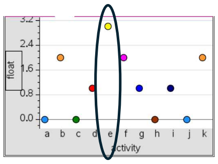
	A ac…	B pr…	C du…	D es	E ef	F su…	G ls	H lf	I float	J
=									=lf−ef	
1	a	no	9	0	9	cd	0	9	0	
2	b	no	11	0	11	fe	2	13	2	
3	c	a	7	9	16	gh	9	16	0	
4	d	a	8	9	17	i	10	18	1	
5	e	b	4	11	15	i	14	18	3	
6	f	b	8	11	19	k	13	21	2	
7	g	c	12	16	28	no	17	29	1	
8	h	c	8	16	24	j	16	24	0	
9	i	de	6	17	23	j	18	24	1	
10	j	hi	5	24	29	no	24	29	0	
11	k	f	8	19	27	no	21	29	2	
12		com…	29							
13										
14										

C12 =max(max(e7,e10),e11)



Critical Path A → C → H → J

b. Which activity can be delayed for the longest time without affecting the minimum completion time of the project?

	Activity E, it has the longest Float Time.

Use the following information to answer parts c – e.

A change is made to the order of activities.

The table below shows the activities and their new latest starting times in weeks.

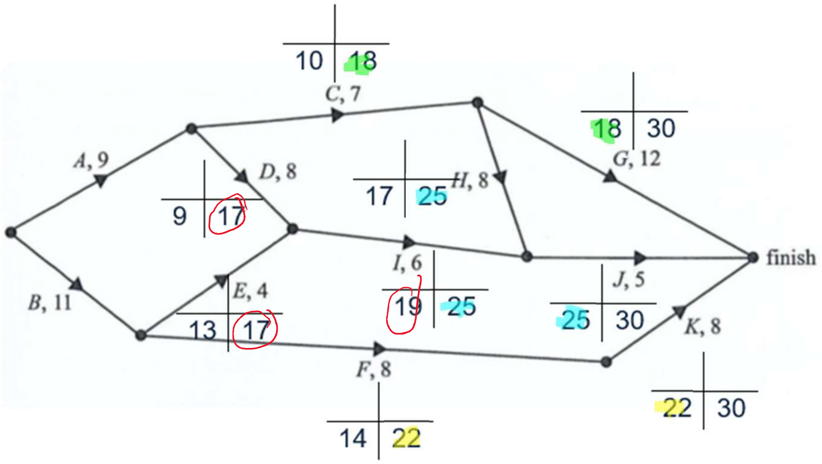
Activity	Latest Starting time (weeks)
A	0
B	2
C	10
D	9
E	13
F	14
G	18
H	17
I	19
J	25
K	22

A dummy activity is now required in the network.

c. On the directed network below, draw a directed edge to represent the dummy activity. Include a label.

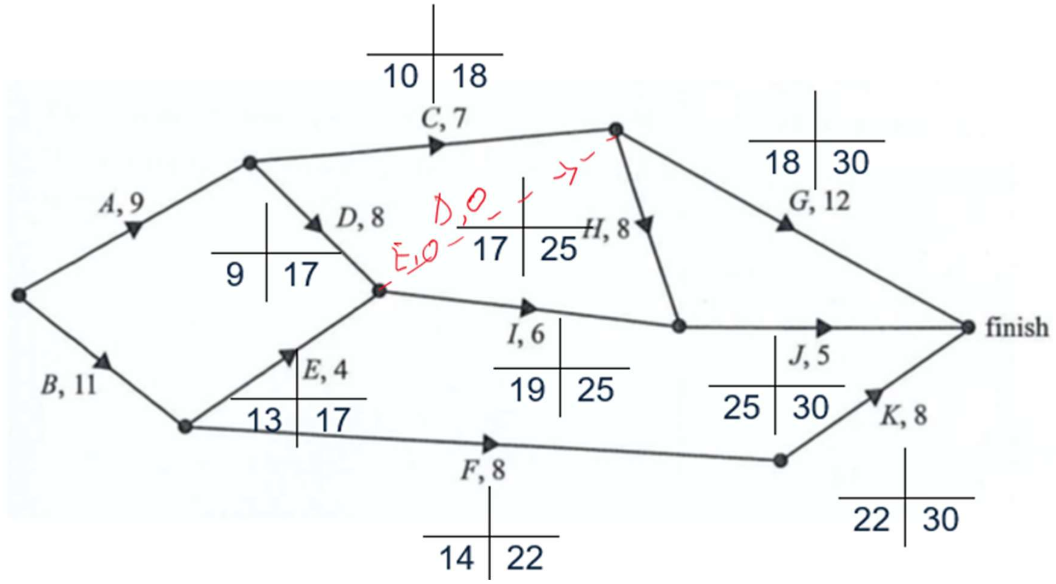
LST is to do with backward scanning. Need to work out new LFT and trace backwards.

Activity	New LST weeks	New LFT = LST + Duration
A	0	0+9=9
B	2	2+11=13
C	10	10+7=18
D	9	9+8=17
E	13	13+4=17
F	14	14+8=22
G	18	18+12=30
H	17	17+8=25
I	19	19+6=25
J	25	25+5=30
K	22	22+8=30



A mismatch is found in D (LFT = 17), E (LFT=17) connect to I (LST = 19). This means both D and E need to connect to an activity with LST = 17 and we saw that it is activity H.

Therefore, draw a dummy activity from end of D (LFT = 17) and E (LFT = 17) to start of H (LST = 17)

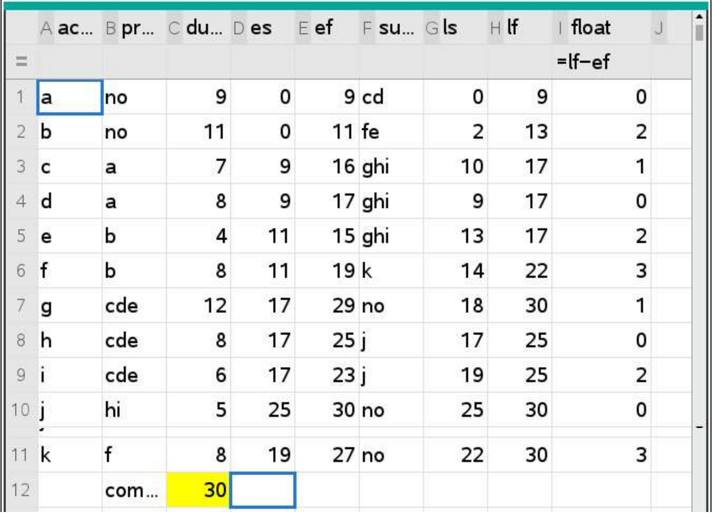


d. What is the new minimum completion time of the project?

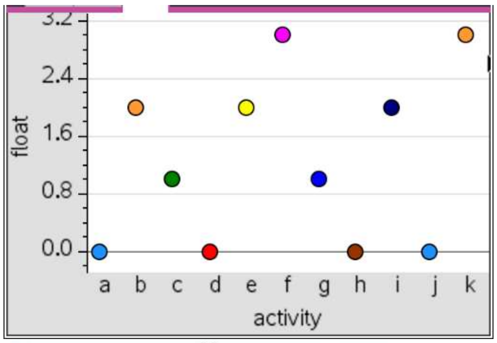
30 weeks (as seen in LFT of activities at finish point)

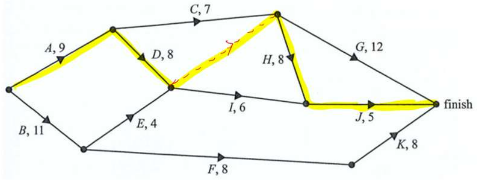
The owners of the supermarket want the project completed earlier.
They will pay to reduce the time of some of the activities.
A reduction in completion time of an activity will incur an additional cost of **$10 000 per week**.
Activities can be reduced by a maximum of **two weeks**.
The minimum number of weeks an activity can be reduced to is **seven weeks**.
What is the minimum amount the owners of the supermarket will have to pay to reduce the completion time of the project as much as possible?

Update Spreadsheet



	A ac...	B pr...	C du...	D es	E ef	F su...	G ls	H lf	I float	J
=									=lf−ef	
1	a	no	9	0	9	cd	0	9	0	
2	b	no	11	0	11	fe	2	13	2	
3	c	a	7	9	16	ghi	10	17	1	
4	d	a	8	9	17	ghi	9	17	0	
5	e	b	4	11	15	ghi	13	17	2	
6	f	b	8	11	19	k	14	22	3	
7	g	cde	12	17	29	no	18	30	1	
8	h	cde	8	17	25	j	17	25	0	
9	i	cde	6	17	23	j	19	25	2	
10	j	hi	5	25	30	no	25	30	0	
11	k	f	8	19	27	no	22	30	3	
12		com...	30							

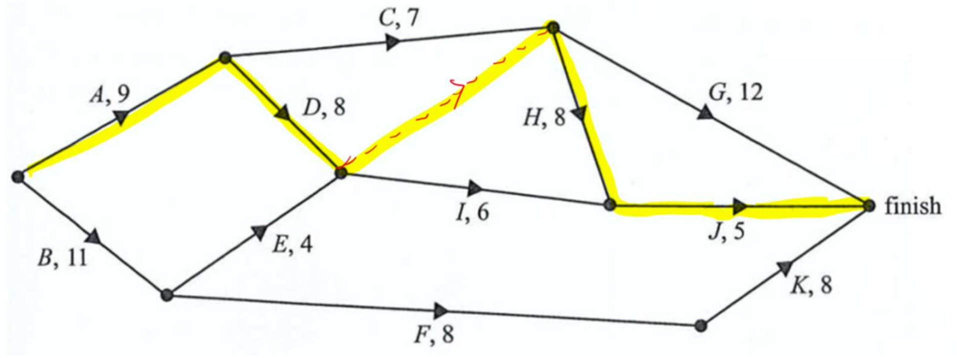




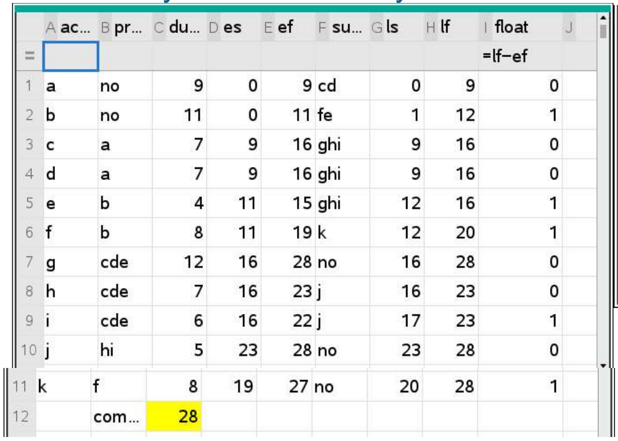
Four activities on Critical Path A (9) + D (8) + H (8) + J (5) = 30 weeks

Observing the critical path, and multiple edges:

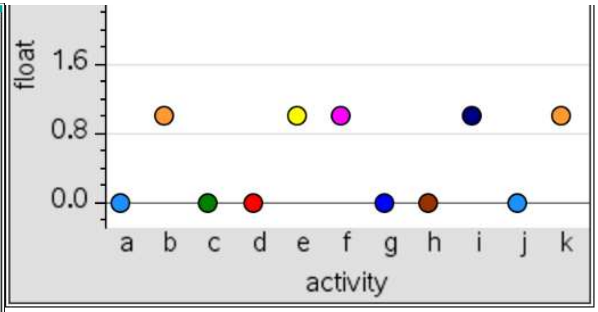
1. C (7), D (8) are two edges from same two vertices. This means we can crash D by 1 week
2. H (8) + J (5) = (13) and G (12) are two edges from same two vertices. This means we can crash either H or J by 1 week. But J < 7, so reduce H.

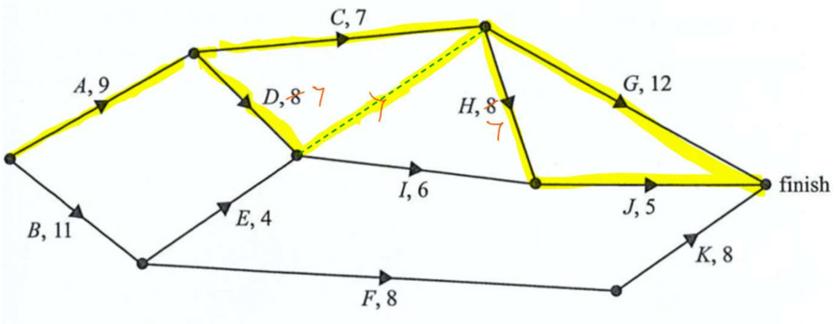


Reduce D by 1 week and H by 1 week:



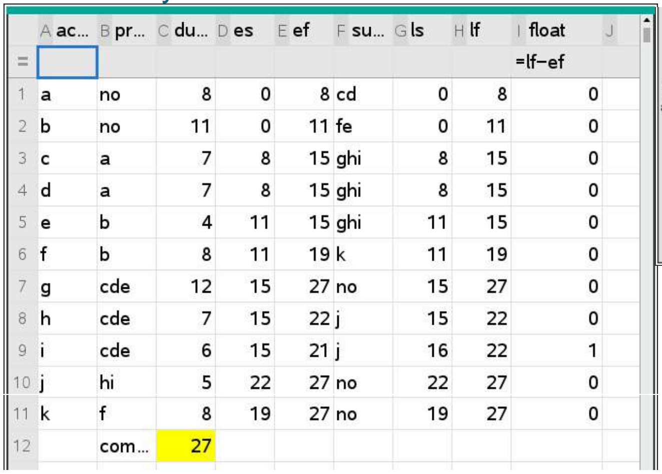
	A ac...	B pr...	C du...	D es	E ef	F su...	G ls	H lf	I float	J
=									=lf−ef	
1	a	no	9	0	9	cd	0	9	0	
2	b	no	11	0	11	fe	1	12	1	
3	c	a	7	9	16	ghi	9	16	0	
4	d	a	7	9	16	ghi	9	16	0	
5	e	b	4	11	15	ghi	12	16	1	
6	f	b	8	11	19	k	12	20	1	
7	g	cde	12	16	28	no	16	28	0	
8	h	cde	7	16	23	j	16	23	0	
9	i	cde	6	16	22	j	17	23	1	
10	j	hi	5	23	28	no	23	28	0	
11	k	f	8	19	27	no	20	28	1	
12		com...	28							



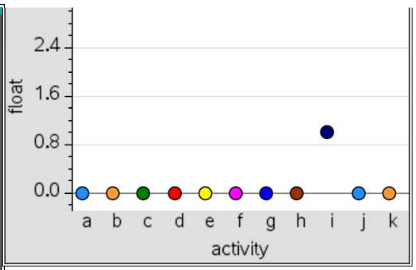


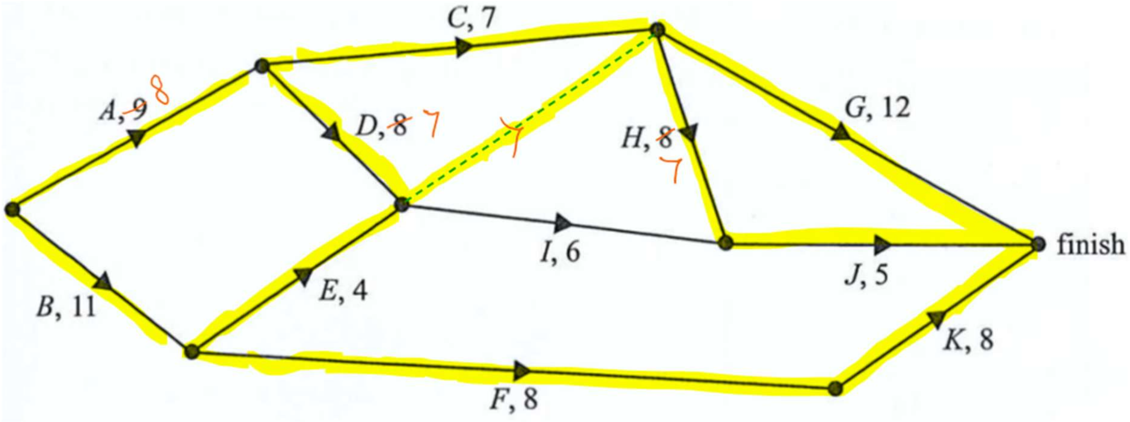
1. D (7) - $10 000, H (7) - $10 000, total $20 000 so far
2. Can't reduce C, D, H further as min weeks of each activity is 7
3. Reduce A (9) by 1 week will reduce completion time further by 1 week.
4. Reduce G (12) by 1 week will not reduce completion time unless also reduce either H or J by 1 week. But this costs $20 000 for reduction of 1 week. (Plus H, J <= 7)

Reduce A by 1 week:



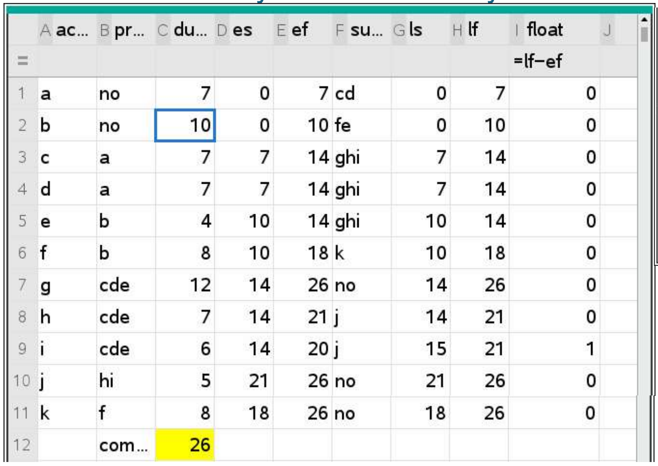
	A ac…	B pr…	C du…	D es	E ef	F su…	G ls	H lf	I float	J
=									=lf−ef	
1	a	no	8	0	8	cd	0	8	0	
2	b	no	11	0	11	fe	0	11	0	
3	c	a	7	8	15	ghi	8	15	0	
4	d	a	7	8	15	ghi	8	15	0	
5	e	b	4	11	15	ghi	11	15	0	
6	f	b	8	11	19	k	11	19	0	
7	g	cde	12	15	27	no	15	27	0	
8	h	cde	7	15	22	j	15	22	0	
9	i	cde	6	15	21	j	16	22	1	
10	j	hi	5	22	27	no	22	27	0	
11	k	f	8	19	27	no	19	27	0	
12		com…	27							



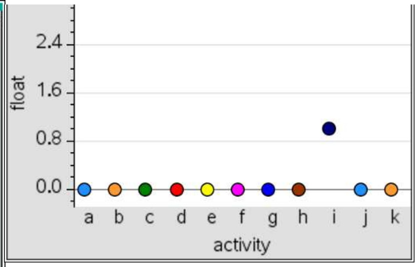


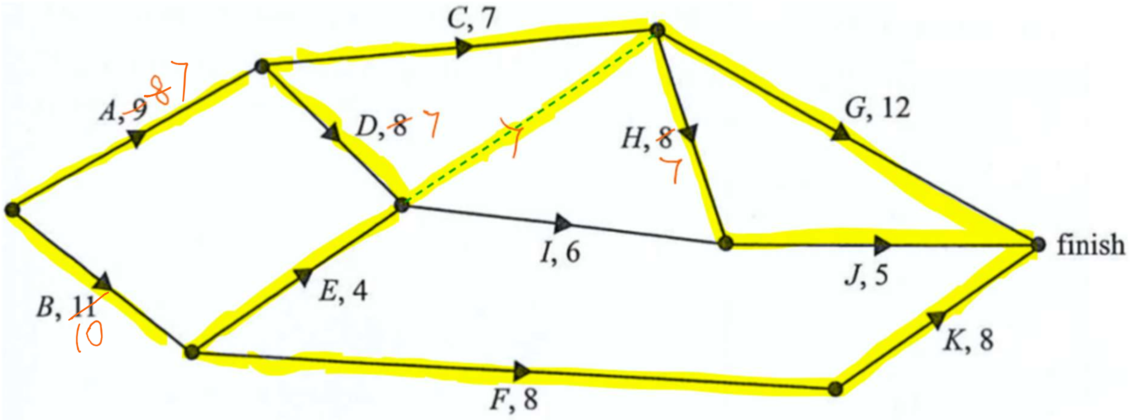
1. A (8) - $10 000, D (7) - $10 000, H (7) - $10 000, total $30 000 so far
2. Can’t reduce C, D, H further as min weeks of each activity is 7
3. A (8) + D (7) = 15 is the same as B (11) + E (4). Reduce both A and B by 1 week

Reduce A further by 1 week and B by 1 week



	A ac...	B pr...	C du...	D es	E ef	F su...	G ls	H lf	I float	J
=									=lf−ef	
1	a	no	7	0	7	cd	0	7	0	
2	b	no	10	0	10	fe	0	10	0	
3	c	a	7	7	14	ghi	7	14	0	
4	d	a	7	7	14	ghi	7	14	0	
5	e	b	4	10	14	ghi	10	14	0	
6	f	b	8	10	18	k	10	18	0	
7	g	cde	12	14	26	no	14	26	0	
8	h	cde	7	14	21	j	14	21	0	
9	i	cde	6	14	20	j	15	21	1	
10	j	hi	5	21	26	no	21	26	0	
11	k	f	8	18	26	no	18	26	0	
12		com...	26							





1. A (7) - $20 000, B(11) - $10 000, D (7) - $10 000, H (7) - $10 000, total $50 000 so far
2. Can't reduce A, C, D, H further as min weeks of each activity is 7
3. Hence, can't reduce B,E as they share same vertices with AD
4. Also, can't reduce G as it shares same vertices with H,J and both can't be reduced.

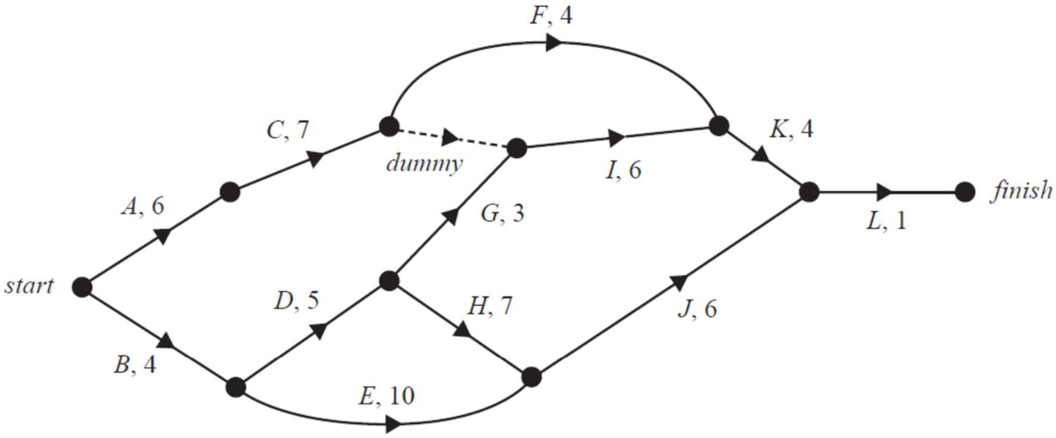
The minimum amount the owners of the supermarket will have to pay to reduce the completion time of the project as much as possible is **$50 000.**

[2023 VCAA General Maths Paper 2 Q 14]

Question 14 (5 marks)

One of the landmarks in state *A* requires a renovation project.

This project involves 12 activities, *A* to *L*. The directed network below shows these activities and their completion times, in days.



The table below shows the 12 activities that need to be completed for the renovation project.

It also shows the earliest start time (EST), the duration, and the immediate predecessors for the activities.

The immediate predecessor(s) for activity *I* and the EST for activity *J* are missing.

Activity	EST	Duration	Immediate predecessor(s)
A	0	6	–
B	0	4	–
C	6	7	*A*
D	4	5	*B*
E	4	10	*B*
F	13	4	*C*
G	9	3	*D*
H	9	7	*D*
I	13	6	
J		6	*E, H*
K	19	4	*F, I*
L	23	1	*J, K*

a. Write down the immediate predecessor(s) for activity *I*. 1 mark

____

b. What is the earliest start time, in days, for activity *J*? 1 mark

____

c. How many activities have a float time of zero? 1 mark

____

The managers of the project are able to reduce the time, in days, of six activities.

These reductions will result in an increase in the cost of completing the activity.

The maximum decrease in time of any activity is two days.

Activity	*A*	*B*	*F*	*H*	*I*	*K*
Daily cost ($)	1500	2000	2500	1000	1500	3000

d. If activities *A* and *B* have their completion time reduced by two days each, the overall completion time of the project will be reduced.

What will be the maximum reduction time, in days? 1 mark

____

____

e. The managers of the project have a maximum budget of $15 000 to reduce the time for several activities to produce the maximum reduction in the project's overall completion time.

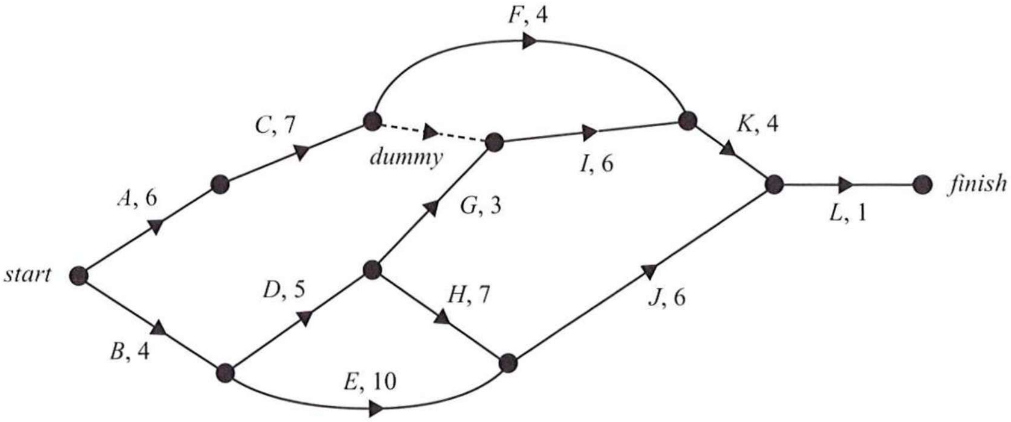
Complete the table below, showing the reductions in individual activity completion times that would achieve the earliest completion time within the $15 000 budget. 1 mark

Activity	**Reduction in completion time (0, 1 or 2 days)**
A	
B	
F	
H	
I	
K	

[2023 VCAA General Maths Paper 2 Q 14]

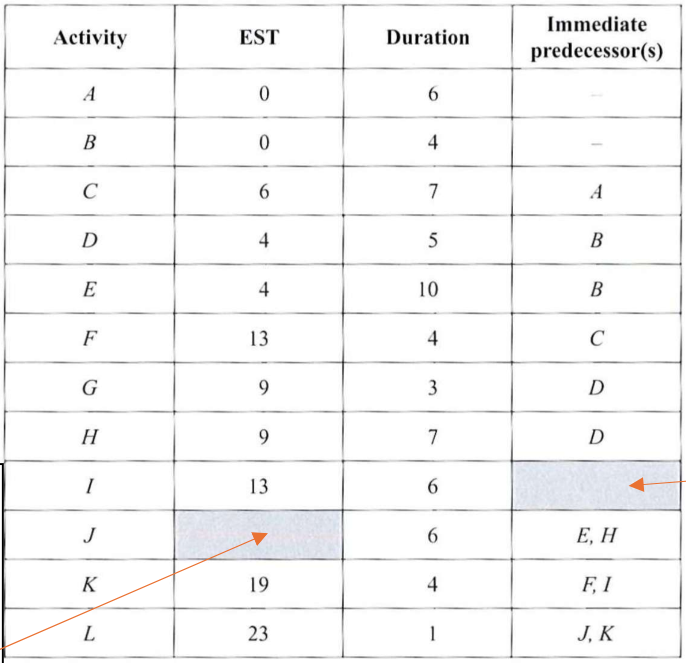
One of the landmarks in state *A* requires a renovation project.

This project involves 12 activities, *A* to *L*. The directed network below shows these activities and their completion times, in days.



The table below shows the 12 activities that need to be completed for the renovation project.

It also shows the earliest start time (EST), the duration, and the immediate predecessors for the activities. The immediate predecessor(s) for activity *I* and the EST for activity *J* are missing.



Activity	EST	Duration	Immediate predecessor(s)
A	0	6	–
B	0	4	–
C	6	7	*A*
D	4	5	*B*
E	4	10	*B*
F	13	4	*C*
G	9	3	*D*
H	9	7	*D*
I	13	6	
J		6	*E*, *H*
K	19	4	*F*, *I*
L	23	1	*J*, *K*

A quick forward scan to J and choose the largest:

B – D – H = 16

B – E = 14

From network above, predecessor of *I* can be quickly identified as

Step One: Copy information from predecessor table to CAS table

	A activity	**B** predecessor	**C** duration	**D** EST	**E** EFT	**F** successor	**G** LST	**H** LFT	**I** FloatTime
=									
1	A	No	6						
2	B	No	4						
3	C	A	7						
4	D	B	5						
5	E	B	10						
6	F	C	4						
7	G	D	3						
8	H	D	7						
9	I	C and G	6						
10	J	E and H	6						
11	K	F and I	4						
12	L	J and K	1						

CAS Useful short cut key

Ctrl 7 – goto top of list

Crtl 1 – goto bottom of list

Step Two: Early Finish Time of each activity = Early Start Time + Duration

In Cell E1, write =b1+c1 and then fill down

	A activity	**B** predecessor	**C** duration	**D** EST	**E** EFT	**F** successor	**G** LST	**H** LFT	**I** FloatTime
=									
1	A	No	6		**=c1+d1**				
2	B	No	4						
3	C	A	7						
4	D	B	5						
5	E	B	10						
6	F	C	4						
7	G	D	3						
8	H	D	7						
9	I	C and G	6						
10	J	E and H	6						
11	K	F and I	4						
12	L	J and K	1						

Step Three: Fill in EST = EFT of predecessor. Use Max( ) for more than one predecessor.

	A activity	**B** predecessor	**C** duration	**D** EST	**E** EFT	**F** successor	**G** LST	**H** LFT	**I** Float Time
=									
1	A	No	6	**0**	**This should be automatically filled as per formula written before**				
2	B	No	4	**0**					
3	C	A	7	**=e1**					
4	D	B	5	**=e2**					
5	E	B	10	**=e2**					
6	F	C	4	**=e3**					
7	G	D	3	**=e4**					
8	H	D	7	**=e4**					
9	I	C and G	6	**=max(e3,e7)**					
10	J	E and H	6	**=max(e5,e8)**					
11	K	F and I	4	**=max(e6,e9)**					
12	L	J and K	1	**=max(e10,e11)**					

Result after Step Three.

	A activity	**B** predecessor	**C** duration	**D** EST	**E** EFT	**F** successor	**G** LST	**H** LFT	**I** FloatTime
=									
1	A	No	6	0	**6**				
2	B	No	4	0	**4**				
3	C	A	7	6	**13**				
4	D	B	5	4	**9**				
5	E	B	10	4	**14**				
6	F	C	4	13	**17**				
7	G	D	3	9	**12**				
8	H	D	7	9	**16**				
9	I	C and G	6	13	**19**				
10	J	E and H	6	16	**22**				
11	K	F and I	4	19	**23**				
12	L	J and K	1	23	**24**			**=e12**	
13		Completion Time	**=e12**						

Step Four: Fill in information of successor (Read from **table** or **network**)

	A activity	**B** predecessor	**C** duration	**D** EST	**E** EFT	**F** successor	**G** LST	**H** LFT	**I** FloatTime
=									
1	A	No	6	0	6	**C**			
2	B	No	4	0	4	**D and E**			
3	C	A	7	6	13	**F and I**			
4	D	B	5	4	9	**G and H**			
5	E	B	10	4	14	**J**			
6	F	C	4	13	17	**K**			
7	G	D	3	9	12	**I**			
8	H	D	7	9	16	**J**			
9	I	C and G	6	13	19	**K**			
10	J	E and H	6	16	22	**L**			
11	K	F and I	4	19	23	**L**			
12	L	J and K	1	23	24	**No**		**24**	
13		**Completion Time**	**24**						

Step Five: LST of each activity = LFT - duration

	A activity	**B** predecessor	**C** duration	**D** EST	**E** EFT	**F** successor	**G** LST	**H** LFT	**I** FloatTime
=									
1	A	No	6	0	6	C	**=h1-c1**		
2	B	No	4	0	4	D and E			
3	C	A	7	6	13	F and I			
4	D	B	5	4	9	G and H			
5	E	B	10	4	14	J			
6	F	C	4	13	17	K			
7	G	D	3	9	12	I			
8	H	D	7	9	16	J			
9	I	C and G	6	13	19	K			
10	J	E and H	6	16	22	L			
11	K	F and I	4	19	23	L			
12	L	J and K	1	23	24	No		**24**	
13		**Completion Time**	**24**						

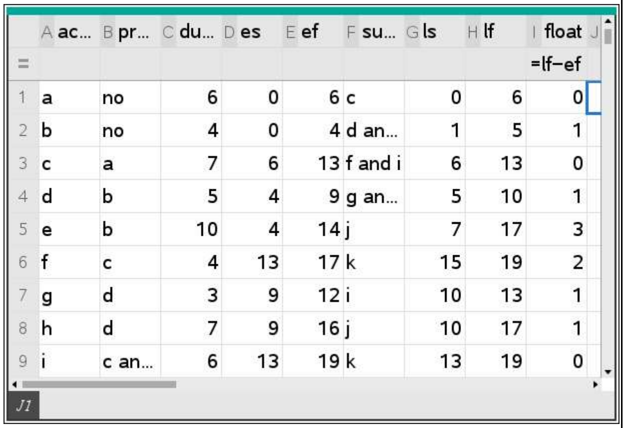
Step Six: LFT of each activity = LST of successor activity. Use min( ) for more than one activity.

	A activity	**B** predecessor	**C** duration	**D** EST	**E** EFT	**F** successor	**G** LST	**H** LFT	**I** FT
=									
1	A	No	6	0	6	C	**This should be automatically filled as per formula written before**	**=g3**	
2	B	No	4	0	4	D and E		**=min(g4,g5)**	
3	C	A	7	6	13	F and I		**=min(g6,g9)**	
4	D	B	5	4	9	G and H		**=min(g7,g8)**	
5	E	B	10	4	14	J		**=g10**	
6	F	C	4	13	17	K		**=g11**	
7	G	D	3	9	12	I		**=g9**	
8	H	D	7	9	16	J		**=g10**	
9	I	C and G	6	13	19	K		**=g11**	
10	J	E and H	6	16	22	L		**=g12**	
11	K	F and I	4	19	23	L		**=g12**	
12	L	J and K	1	23	24	No		**24**	
13		**Completion Time**	24						

Step Seven: Generate Float Time of each activity, Float Time = LFT - EFT

	A activity	**B** predecessor	**C** duration	**D** EST	**E** EFT	**F** successor	**G** LST	**H** LFT	**I** FloatTime
=									**=LFT-EFT**
1	A	No	6	0	6	C	**0**	**6**	**0**
2	B	No	4	0	4	D and E	**1**	**5**	**1**
3	C	A	7	6	13	F and I	**6**	**13**	**0**
4	D	B	5	4	9	G and H	**5**	**10**	**1**
5	E	B	10	4	14	J	**7**	**17**	**3**
6	F	C	4	13	17	K	**15**	**19**	**2**
7	G	D	3	9	12	I	**10**	**13**	**1**
8	H	D	7	9	16	J	**10**	**17**	**1**
9	I	C and G	6	13	19	K	**13**	**19**	**0**
10	J	E and H	6	16	22	L	**17**	**23**	**1**
11	K	F and I	4	19	23	L	**19**	**23**	**0**
12	L	J and K	1	23	24	No	**23**	**24**	**0**
13		**Completion Time**	24						

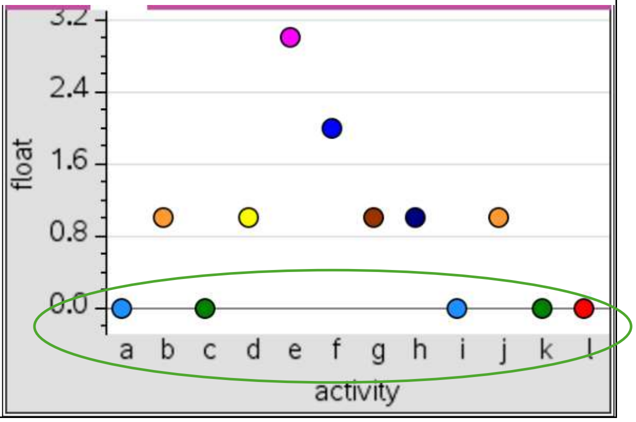
Screen shot of table on CAS



	A ac...	B pr...	C du...	D es	E ef	F su...	G ls	H lf	I float	J
=									=lf−ef	
1	a	no	6	0	6	c	0	6	0	
2	b	no	4	0	4	d an...	1	5	1	
3	c	a	7	6	13	f and i	6	13	0	
4	d	b	5	4	9	g an...	5	10	1	
5	e	b	10	4	14	j	7	17	3	
6	f	c	4	13	17	k	15	19	2	
7	g	d	3	9	12	i	10	13	1	
8	h	d	7	9	16	j	10	17	1	
9	i	c an...	6	13	19	k	13	19	0	

J1

Plot 'Data and Statistics' page to see Critical activities. Float time = 0 A – C – I – K - L

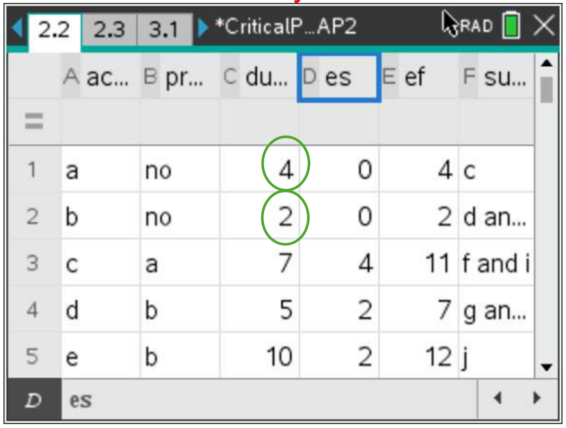


Question 14d

If activities *A* and *B* have their completion time reduced by two days each, the overall completion time of the project will be reduced.

What will be the maximum reduction time, in days? 24 – 22 = 2 days

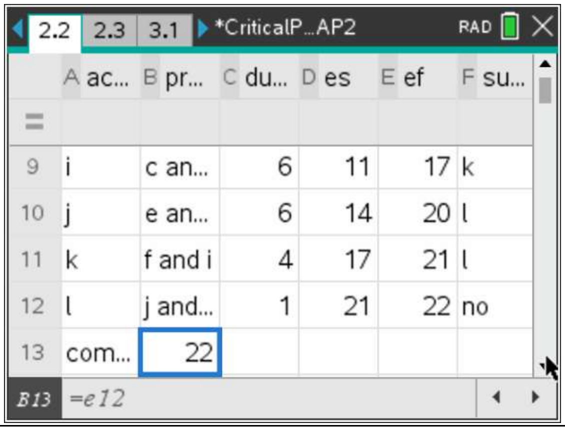
Change duration of Activity A from 6 to 4 and duration of Activity *B* from 4 to 2



	A ac...	B pr...	C du...	D es	E ef	F su...
=						
1	a	no	4	0	4	c
2	b	no	2	0	2	d an...
3	c	a	7	4	11	f and i
4	d	b	5	2	7	g an...
5	e	b	10	2	12	j

D es

Press Crtl , 1 to go to the bottom of information and see completion time updated to 22



	A ac...	B pr...	C du...	D es	E ef	F su...
=						
9	i	c an...	6	11	17	k
10	j	e an...	6	14	20	l
11	k	f and i	4	17	21	l
12	l	j and...	1	21	22	no
13	com...	22				

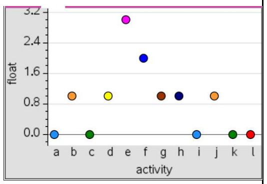
B13 =e12

Question 14e The managers of the project have a maximum budget of $15000 to reduce the time for several activities to produce the maximum reduction in the project's overall completion time. Complete the table below, showing the reductions in individual activity completion times that would achieve the earliest completion time within the $15000 budget.

The reduction will result in an increase in the cost of completing the activity as per given table:

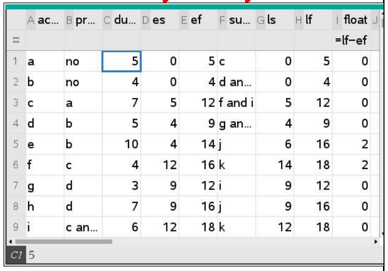
Activity	A	B	F	H	I	K
Daily Cost ($)	1500	2000	2500	1000	1500	3000

Start with identifying Critical Activities A, C, I, K, L



A, I, K can be crashed with A or I the cheapest

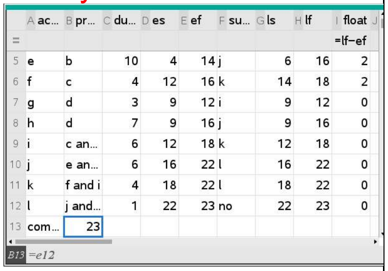
Reduce A by 1 day - $1500



	A ac...	B pr...	C du...	D es	E ef	F su...	G ls	H lf	I float
=									=lf−ef
1	a	no	5	0	5	c	0	5	0
2	b	no	4	0	4	d an...	0	4	0
3	c	a	7	5	12	f and i	5	12	0
4	d	b	5	4	9	g an...	4	9	0
5	e	b	10	4	14	j	6	16	2
6	f	c	4	12	16	k	14	18	2
7	g	d	3	9	12	i	9	12	0
8	h	d	7	9	16	j	9	16	0
9	i	c an...	6	12	18	k	12	18	0

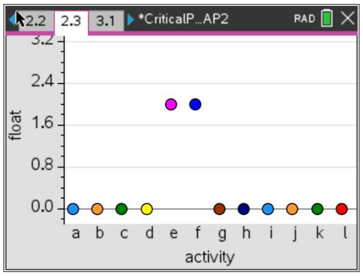
C1 5

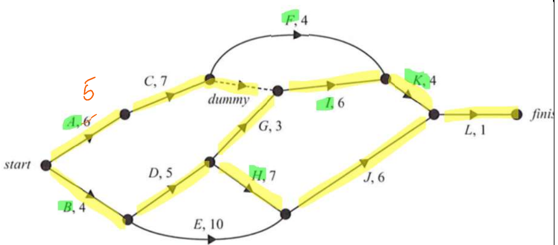
Completion time becomes 23 days



	A ac...	B pr...	C du...	D es	E ef	F su...	G ls	H lf	I float
=									=lf−ef
5	e	b	10	4	14	j	6	16	2
6	f	c	4	12	16	k	14	18	2
7	g	d	3	9	12	i	9	12	0
8	h	d	7	9	16	j	9	16	0
9	i	c an...	6	12	18	k	12	18	0
10	j	e an...	6	16	22	l	16	22	0
11	k	f and i	4	18	22	l	18	22	0
12	l	j and...	1	22	23	no	22	23	0
13	com...	23							

B13 =e12





Notice the multiple critical paths

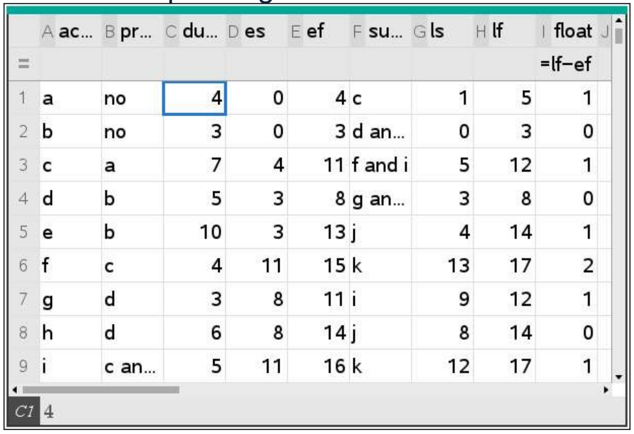
A - C = B – D – G =12 days

Crash A by 1 day requires crash B by 1 day

G – I – K = H – J = 13 days to complete

Crash H by 1 day requires crash I or K by 1 day but I is cheaper so choose I

Enter corresponding reduction to CAS table



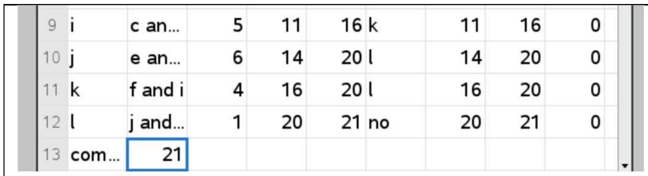
	A ac...	B pr...	C du...	D es	E ef	F su...	G ls	H lf	I float
=									=lf−ef
1	a	no	4	0	4	c	1	5	1
2	b	no	3	0	3	d an...	0	3	0
3	c	a	7	4	11	f and i	5	12	1
4	d	b	5	3	8	g an...	3	8	0
5	e	b	10	3	13	j	4	14	1
6	f	c	4	11	15	k	13	17	2
7	g	d	3	8	11	i	9	12	1
8	h	d	6	8	14	j	8	14	0
9	i	c an...	5	11	16	k	12	17	1

C1 4

Budget: $15 000 not reached

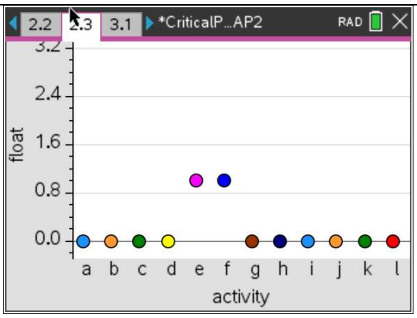
Activity	Reduction	Cost ($)
A	2	1500 x 2
B	1	2000
H	1	1000
I	1	1500
	Total	7 500

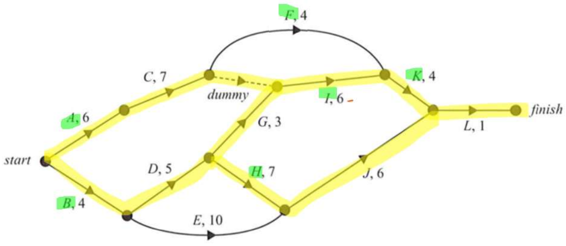
Check Critical Path



9	i	c an...	5	11	16	k	11	16	0
10	j	e an...	6	14	20	l	14	20	0
11	k	f and i	4	16	20	l	16	20	0
12	l	j and...	1	20	21	no	20	21	0
13	com...	21							

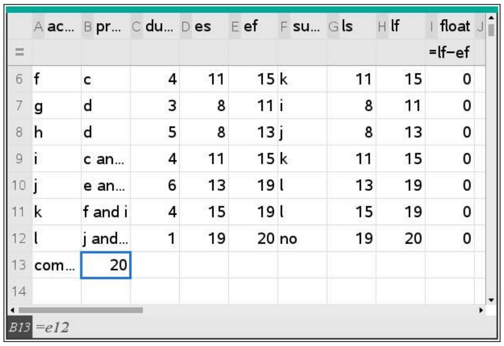
Completion time reduced to 21 days





Reduce H and I by 1 day

Reduce duration of H further by 1 day



	A ac...	B pr...	C du...	D es	E ef	F su...	G ls	H lf	I float
=									=lf−ef
6	f	c	4	11	15	k	11	15	0
7	g	d	3	8	11	i	8	11	0
8	h	d	5	8	13	j	8	13	0
9	i	c an...	4	11	15	k	11	15	0
10	j	e an...	6	13	19	l	13	19	0
11	k	f and i	4	15	19	l	15	19	0
12	l	j and...	1	19	20	no	19	20	0
13	com...	20							
14									

B13 =e12

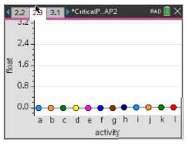
Completion time down to 20 days

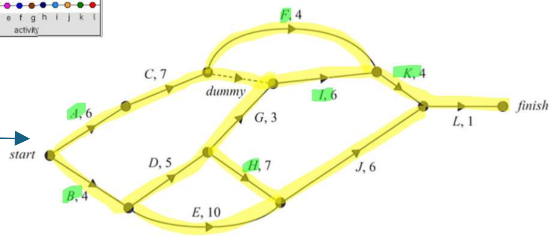
A, H, I have reached 2 days limits so can't reduce further.
B is the cheapest. But if B is reduced by 1, either F or K must be reduced by 1.

Budget: $15 000 not reached

Activity	Reduction	Cost ($)
A	2	1500 x 2
B	1	2000
H	2	1000 x 2
I	2	1500 x 2
	Total	10 000

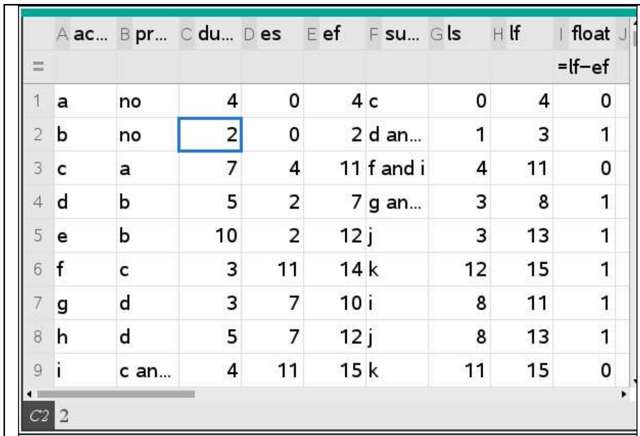
Check Critical Path





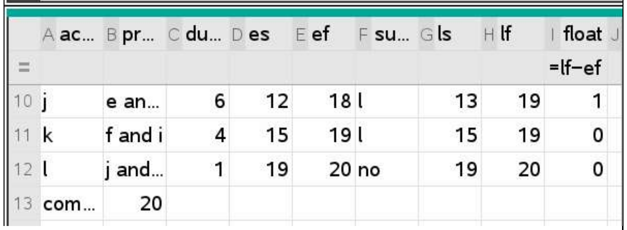
Reduce B by 1 and F by 1 does not reduce completion time further. It stays at 20 days

Reduce B by 1 and K by 1 does reduce completion time further. It comes down to 19 days

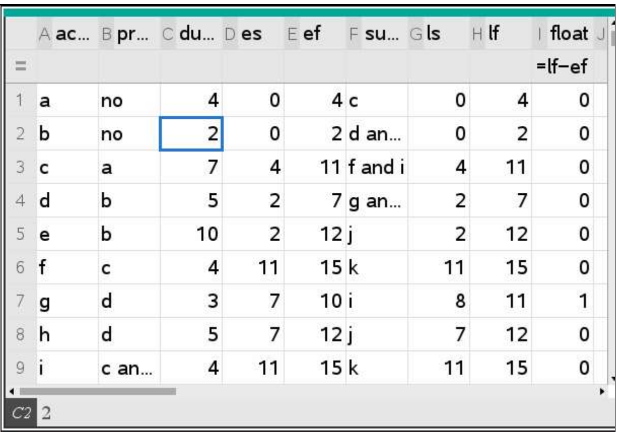


	A ac…	B pr…	C du…	D es	E ef	F su…	G ls	H lf	I float	J
=									=lf−ef	
1	a	no	4	0	4	c	0	4	0	
2	b	no	2	0	2	d an…	1	3	1	
3	c	a	7	4	11	f and i	4	11	0	
4	d	b	5	2	7	g an…	3	8	1	
5	e	b	10	2	12	j	3	13	1	
6	f	c	3	11	14	k	12	15	1	
7	g	d	3	7	10	i	8	11	1	
8	h	d	5	7	12	j	8	13	1	
9	i	c an…	4	11	15	k	11	15	0	

C2 2

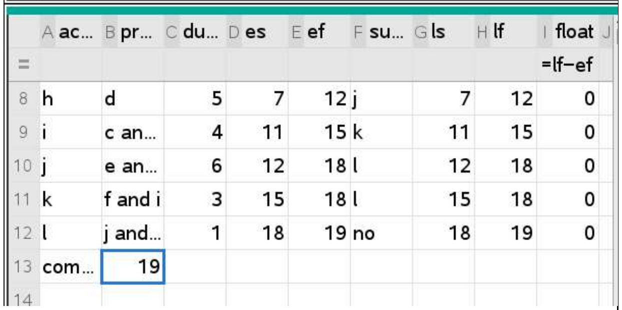


	A ac…	B pr…	C du…	D es	E ef	F su…	G ls	H lf	I float	J
=									=lf−ef	
10	j	e an…	6	12	18	l	13	19	1	
11	k	f and i	4	15	19	l	15	19	0	
12	l	j and…	1	19	20	no	19	20	0	
13	com…	20								



	A ac…	B pr…	C du…	D es	E ef	F su…	G ls	H lf	I float	J
=									=lf−ef	
1	a	no	4	0	4	c	0	4	0	
2	b	no	2	0	2	d an…	0	2	0	
3	c	a	7	4	11	f and i	4	11	0	
4	d	b	5	2	7	g an…	2	7	0	
5	e	b	10	2	12	j	2	12	0	
6	f	c	4	11	15	k	11	15	0	
7	g	d	3	7	10	i	8	11	1	
8	h	d	5	7	12	j	7	12	0	
9	i	c an…	4	11	15	k	11	15	0	

C2 2



	A ac…	B pr…	C du…	D es	E ef	F su…	G ls	H lf	I float	J
=									=lf−ef	
8	h	d	5	7	12	j	7	12	0	
9	i	c an…	4	11	15	k	11	15	0	
10	j	e an…	6	12	18	l	12	18	0	
11	k	f and i	3	15	18	l	15	18	0	
12	l	j and…	1	18	19	no	18	19	0	
13	com…	19								
14										

And
Budget: $15 000 is reached

Activity	Reduction	Cost ($)
A	2	1500 x 2
B	2	2000 x 2
H	2	1000 x 2
I	2	1500 x 2
K	1	3000
	Total	15 000

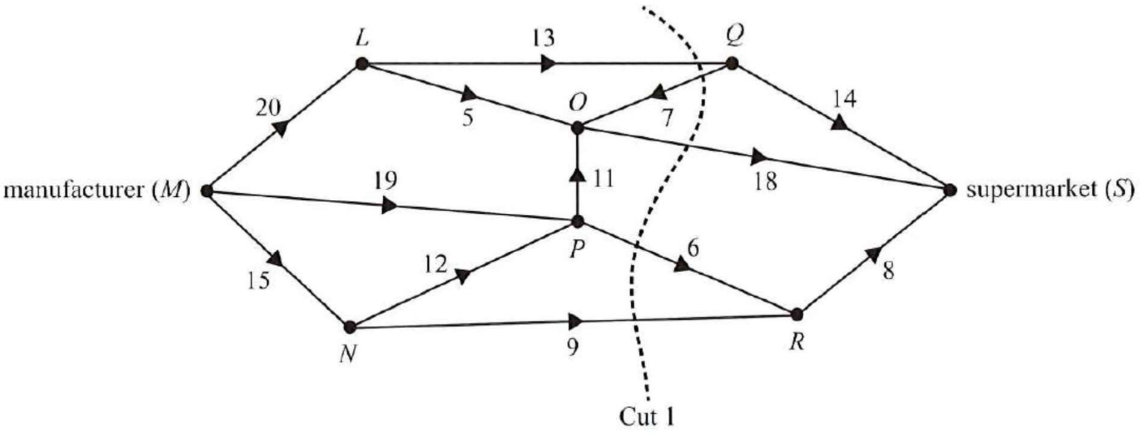
Session B – Flow Problems

[2024 VCAA General Maths Paper 2 Question 14]

Question 14 (3 marks)

A manufacturer (M) makes deliveries to the supermarket (S) via a number of storage warehouses, L, N, O, P, Q and R. These eight locations are represented as vertices in the network below.

The numbers on the edges represent the maximum number of deliveries that can be made between these locations each day.



a. When considering the possible flow of deliveries through this network, many different cuts can be made.

Determine the capacity of Cut 1, shown above. 1 mark

b. Determine the maximum number of deliveries that can be made each day from the manufacturer to the supermarket. 1 mark

c. The manufacturer wants to increase the number of deliveries to the supermarket.

This can be achieved by increasing the number of deliveries between one pair of locations.

Complete the following sentence by writing the locations in the boxes provided:

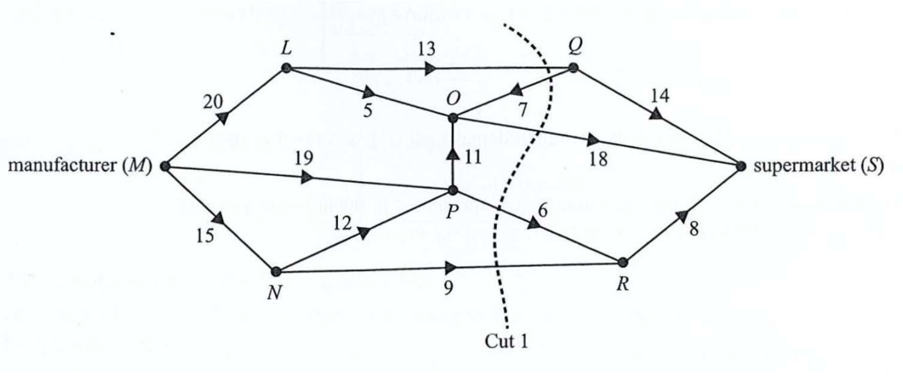
To maximise this increase, the number of deliveries should be increased between

locations ☐ and ☐. 1 mark

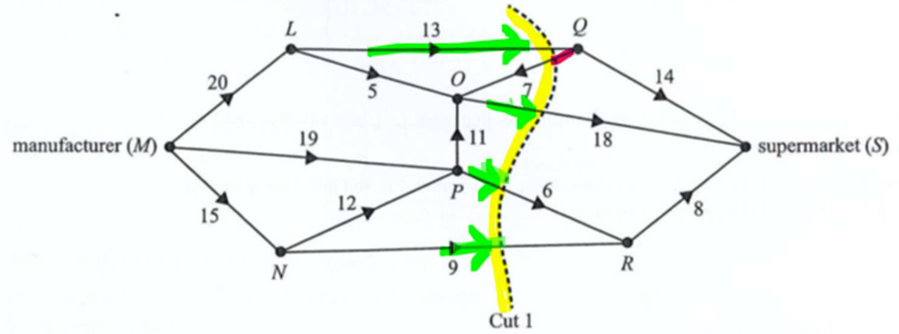
[2024 VCAA General Maths Paper 2 Question 14]

A manufacturer (*M*) makes deliveries to the supermarket (*S*) via a number of storage warehouses, *L*,*N*, *O*, *P*, *Q* and *R*. These eight locations are represented as vertices in the network below.

The numbers on the edges represent the maximum number of deliveries that can be made between these locations each day.

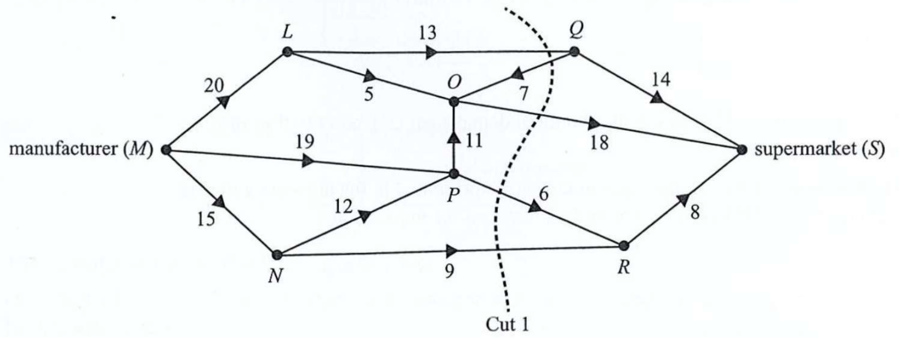


a) When considering the possible flow of deliveries through this network, many different cuts can be made. Determine the capacity of Cut 1, shown above.

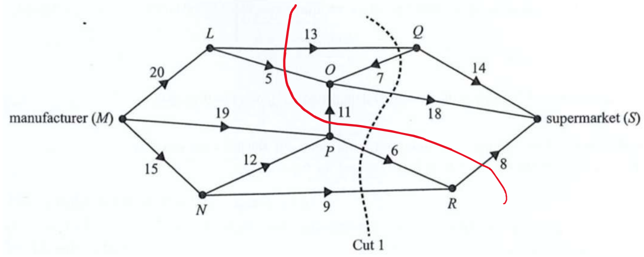


Capacity of Cut 1 = 13 + 18 + 6 + 9 = 46 delivers / day	To determine the capacity of a cut, first highlight the "source" side of the cut. Include only the edges where the flow direction moves from the highlighted (source) side to the non-highlighted (sink) side. These edges (e.g., L→Q, O →S, P→R, and N→R) contribute to the capacity of the cut. The edge Q→O does not contribute to the capacity of the cut because it does not facilitate flow from the source to the sink. When edges L→Q, O →S, P→R, and N→R are cut, no flow from the source to the sink is possible.

b) Determine the maximum number of deliveries that can be made each day from the manufacturer to the supermarket.



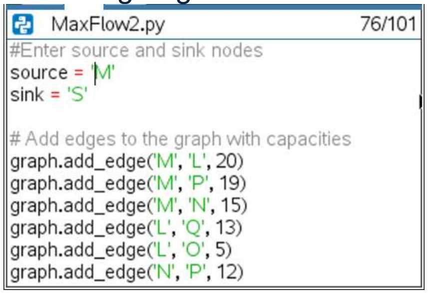
Method 1
Maximum Flow – Minimum Cut



13 + 5 + 11 + 8 = 37 deliveries / day

Method 2
Ford-Fulkerson's Algorithm

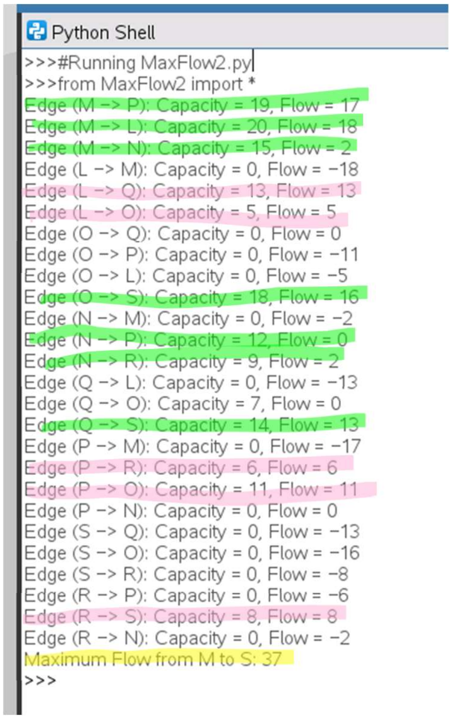
Press Crt-G to change the parameters according to given network



```
#Enter source and sink nodes
source = 'M'
sink = 'S'

# Add edges to the graph with capacities
graph.add_edge('M', 'L', 20)
graph.add_edge('M', 'P', 19)
graph.add_edge('M', 'N', 15)
graph.add_edge('L', 'Q', 13)
graph.add_edge('L', 'O', 5)
graph.add_edge('N', 'P', 12)
```

Press Ctrl – R to run program



```
>>>#Running MaxFlow2.py
>>>from MaxFlow2 import *
Edge (M −> P): Capacity = 19, Flow = 17
Edge (M −> L): Capacity = 20, Flow = 18
Edge (M −> N): Capacity = 15, Flow = 2
Edge (L −> M): Capacity = 0, Flow = −18
Edge (L −> Q): Capacity = 13, Flow = 13
Edge (L −> O): Capacity = 5, Flow = 5
Edge (O −> Q): Capacity = 0, Flow = 0
Edge (O −> P): Capacity = 0, Flow = −11
Edge (O −> L): Capacity = 0, Flow = −5
Edge (O −> S): Capacity = 18, Flow = 16
Edge (N −> M): Capacity = 0, Flow = −2
Edge (N −> P): Capacity = 12, Flow = 0
Edge (N −> R): Capacity = 9, Flow = 2
Edge (Q −> L): Capacity = 0, Flow = −13
Edge (Q −> O): Capacity = 7, Flow = 0
Edge (Q −> S): Capacity = 14, Flow = 13
Edge (P −> M): Capacity = 0, Flow = −17
Edge (P −> R): Capacity = 6, Flow = 6
Edge (P −> O): Capacity = 11, Flow = 11
Edge (P −> N): Capacity = 0, Flow = 0
Edge (S −> Q): Capacity = 0, Flow = −13
Edge (S −> O): Capacity = 0, Flow = −16
Edge (S −> R): Capacity = 0, Flow = −8
Edge (R −> P): Capacity = 0, Flow = −6
Edge (R −> S): Capacity = 8, Flow = 8
Edge (R −> N): Capacity = 0, Flow = −2
Maximum Flow from M to S: 37
>>>
```

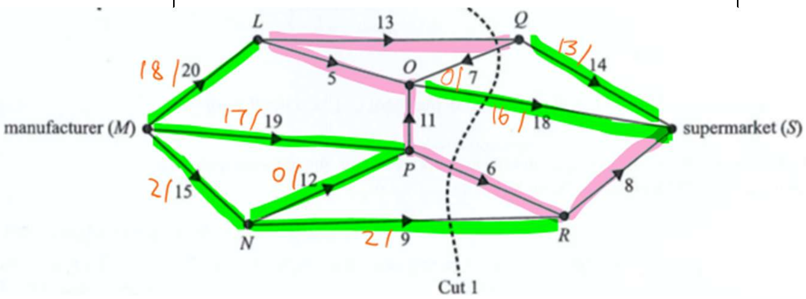
Answer on the last line of the program.
In this case, Maximum flow
= 37 delivers / day

The program gives more than just an answer.
It also gives an optimal flow diagram for further analysis.

See highlighted in red are the saturated pipes (edges) where Capacity = Flow

Ignore the ones with negative Flow

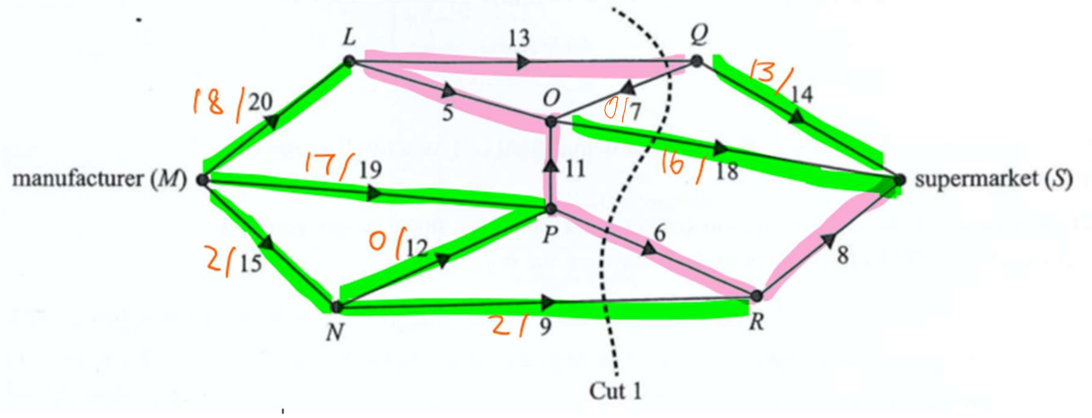
Highlighted in green are pipes (edges) where Capacity > Flow, indicating capacity of edge has not been fully utilized.



c) The manufacturer wants to increase the number of deliveries to the supermarket. This can be achieved by increasing the number of deliveries between one pair of locations.

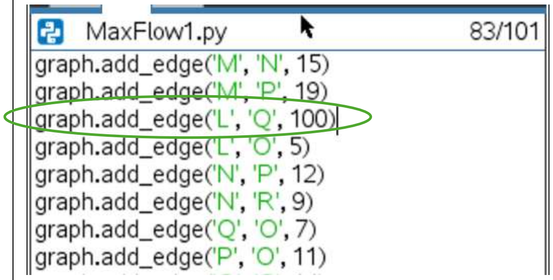
Complete the following sentence by writing the locations in the boxes provided:

To maximise this increase, the number of deliveries should be increased between locations _____ and _____.

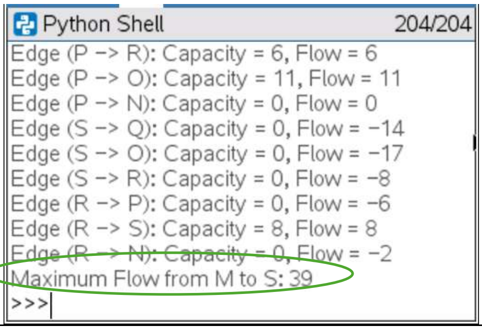


- To increase the maximum flow of a network by increasing capacity of pipes (edges), one or more of the capacity of the saturated pipe must increase.
- Hence, consider the red (saturated) pipes:

1. LQ has a capacity of 13 currently. If its capacity can be increased, it makes no difference whether it increases by 2 or more. This is because pipe ML has only 20-18 = 2 units of extra capacity. While 1 unit can go through QS, the other can go through QO then OS. So Max flow increases from 37 to 39 units.

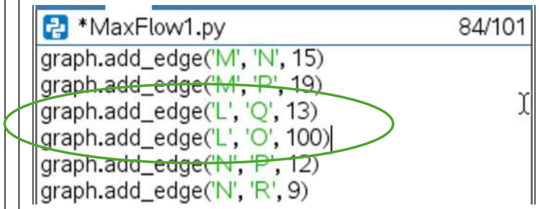


```
graph.add_edge('M', 'N', 15)
graph.add_edge('M', 'P', 19)
graph.add_edge('L', 'Q', 100)
graph.add_edge('L', 'O', 5)
graph.add_edge('N', 'P', 12)
graph.add_edge('N', 'R', 9)
graph.add_edge('Q', 'O', 7)
graph.add_edge('P', 'O', 11)
```

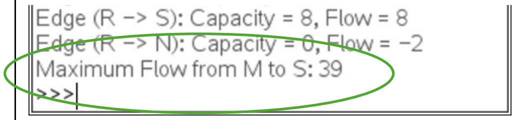


```
Edge (P -> R): Capacity = 6, Flow = 6
Edge (P -> O): Capacity = 11, Flow = 11
Edge (P -> N): Capacity = 0, Flow = 0
Edge (S -> Q): Capacity = 0, Flow = -14
Edge (S -> O): Capacity = 0, Flow = -17
Edge (S -> R): Capacity = 0, Flow = -8
Edge (R -> P): Capacity = 0, Flow = -6
Edge (R -> S): Capacity = 8, Flow = 8
Edge (R -> N): Capacity = 0, Flow = -2
Maximum Flow from M to S: 39
>>>
```

2. Similarly, it makes no difference if LO is increased by 2 or more. So Max Flow also increases from 37 to 39 units.

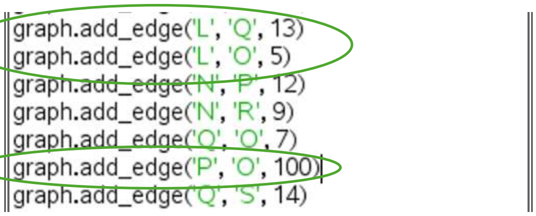


```
graph.add_edge('M', 'N', 15)
graph.add_edge('M', 'P', 19)
graph.add_edge('L', 'Q', 13)
graph.add_edge('L', 'O', 100)
graph.add_edge('N', 'P', 12)
graph.add_edge('N', 'R', 9)
```

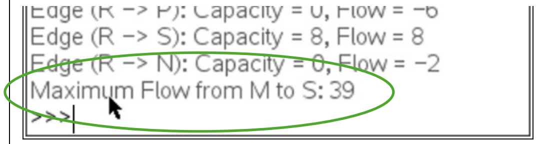


```
Edge (R -> S): Capacity = 8, Flow = 8
Edge (R -> N): Capacity = 0, Flow = -2
Maximum Flow from M to S: 39
>>>
```

3. Similarly, it makes no difference if PO is increased by 2 or more. So Max Flow also increases from 37 to 39 units.

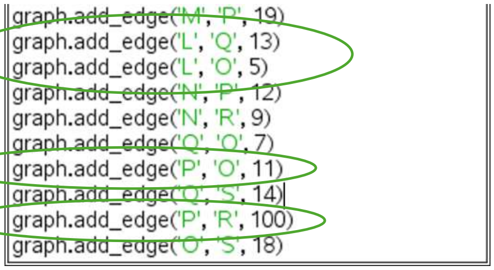


```
graph.add_edge('L', 'Q', 13)
graph.add_edge('L', 'O', 5)
graph.add_edge('N', 'P', 12)
graph.add_edge('N', 'R', 9)
graph.add_edge('Q', 'O', 7)
graph.add_edge('P', 'O', 100)
graph.add_edge('Q', 'S', 14)
```

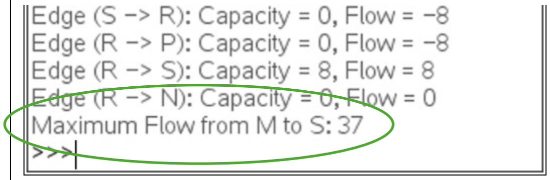


```
Edge (R -> P): Capacity = 0, Flow = −6
Edge (R -> S): Capacity = 8, Flow = 8
Edge (R -> N): Capacity = 0, Flow = −2
Maximum Flow from M to S: 39
>>>
```

4. For PR, it makes no difference whether it is increased or not as the saturated pipe RS shall limit any extra flow from PR.

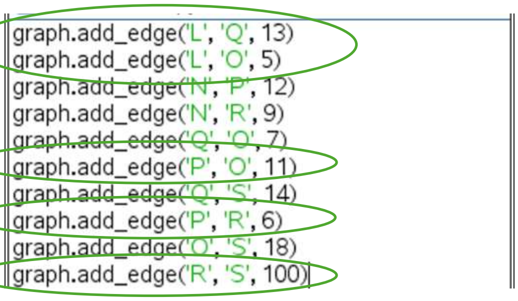


```
graph.add_edge('M', 'P', 19)
graph.add_edge('L', 'Q', 13)
graph.add_edge('L', 'O', 5)
graph.add_edge('N', 'P', 12)
graph.add_edge('N', 'R', 9)
graph.add_edge('Q', 'O', 7)
graph.add_edge('P', 'O', 11)
graph.add_edge('Q', 'S', 14)
graph.add_edge('P', 'R', 100)
graph.add_edge('O', 'S', 18)
```

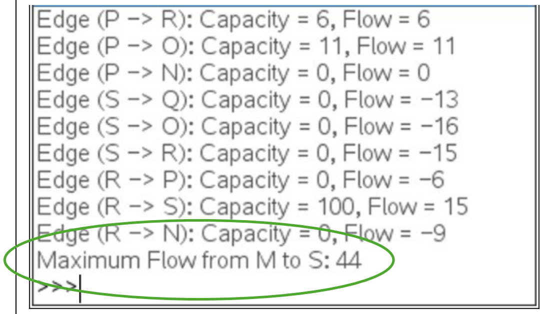


```
Edge (S -> R): Capacity = 0, Flow = −8
Edge (R -> P): Capacity = 0, Flow = −8
Edge (R -> S): Capacity = 8, Flow = 8
Edge (R -> N): Capacity = 0, Flow = 0
Maximum Flow from M to S: 37
>>>
```

5. RS can be increased by 9-2 (Pipe NR) = 7 unit to 15 unit and overall increase Max flow to 37 + 7 = 44 unit



```
graph.add_edge('L', 'Q', 13)
graph.add_edge('L', 'O', 5)
graph.add_edge('N', 'P', 12)
graph.add_edge('N', 'R', 9)
graph.add_edge('Q', 'O', 7)
graph.add_edge('P', 'O', 11)
graph.add_edge('Q', 'S', 14)
graph.add_edge('P', 'R', 6)
graph.add_edge('O', 'S', 18)
graph.add_edge('R', 'S', 100)
```

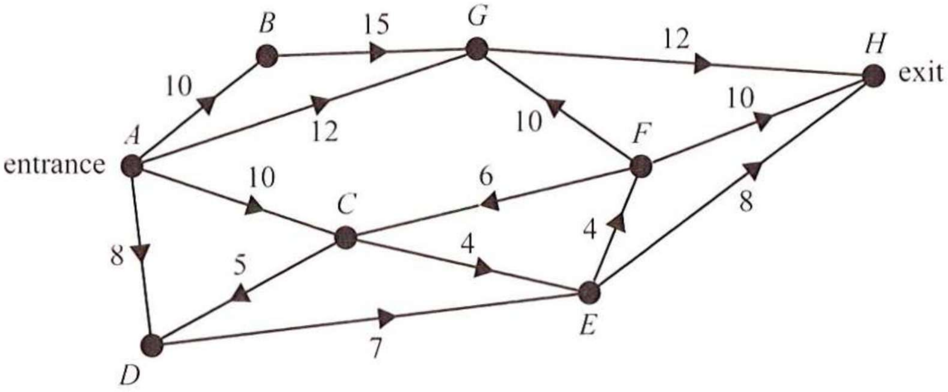


```
Edge (P -> R): Capacity = 6, Flow = 6
Edge (P -> O): Capacity = 11, Flow = 11
Edge (P -> N): Capacity = 0, Flow = 0
Edge (S -> Q): Capacity = 0, Flow = −13
Edge (S -> O): Capacity = 0, Flow = −16
Edge (S -> R): Capacity = 0, Flow = −15
Edge (R -> P): Capacity = 0, Flow = −6
Edge (R -> S): Capacity = 100, Flow = 15
Edge (R -> N): Capacity = 0, Flow = −9
Maximum Flow from M to S: 44
>>>
```

Therefore, to maximise this increase, the number of deliveries should be increased between locations **R** and **S.**

[2023 VCAA General Maths Paper 1 Q 39 and Q 40] – saturated paths (manual calculation)

The network below shows the one-way paths between the entrance, *A*, and the exit, *H,* of a children's maze. The vertices represent the intersections of the one-way paths. The number on each edge is the maximum number of children who are allowed to travel along that path per minute.

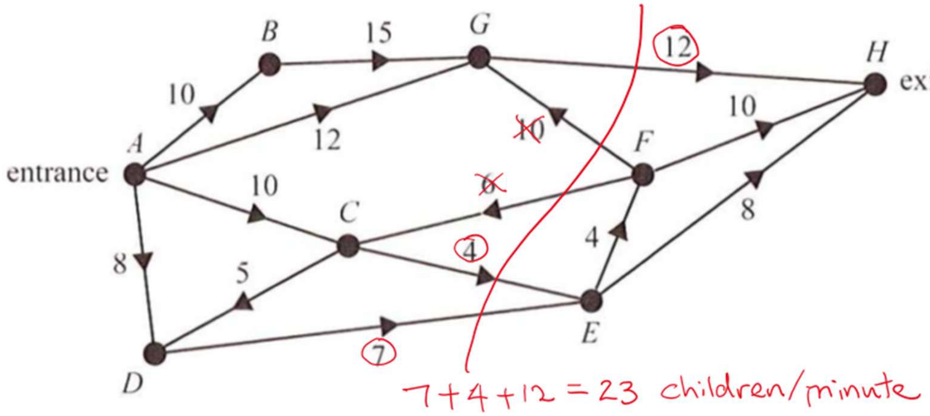


Question 39

Cuts on this network are used to consider the possible flow of children through the maze. The capacity of the minimum cut would be

A. 20 B. 23 C. 24 D. 29 E. 30

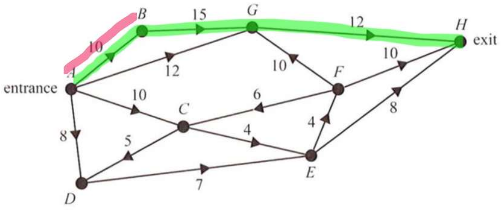
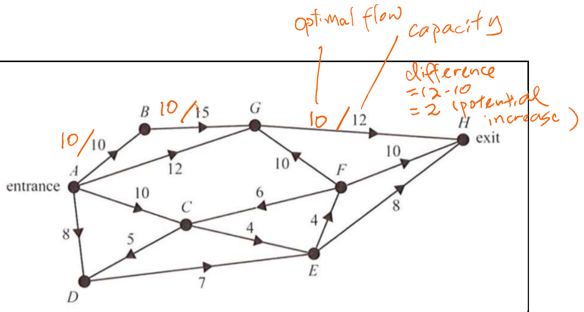
Method 1 – Maximum-flow-minimum-cut

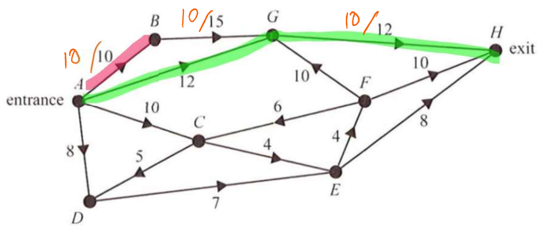
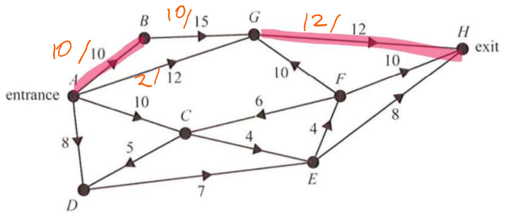


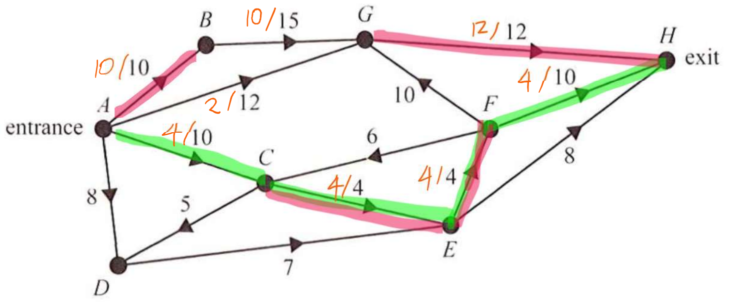
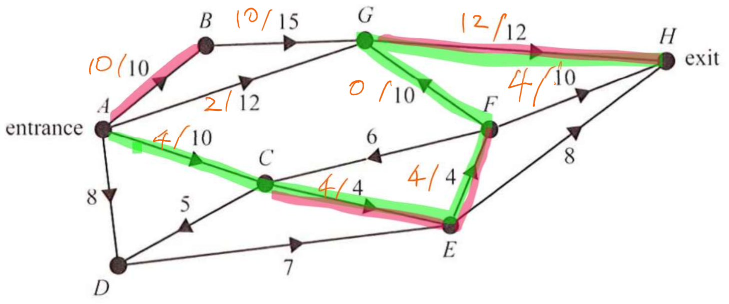
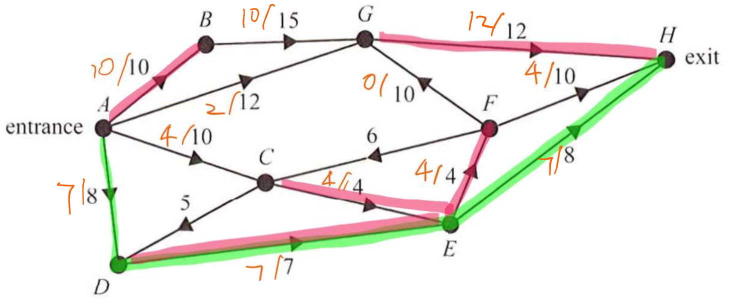
7 + 4 + 12 = 23 children/minute

- If student has good numerical sense, could be the fastest method	- Need to keep exploring possible minimum cut. (add → compare → find next min?)

Method 2 – Saturated paths analysis

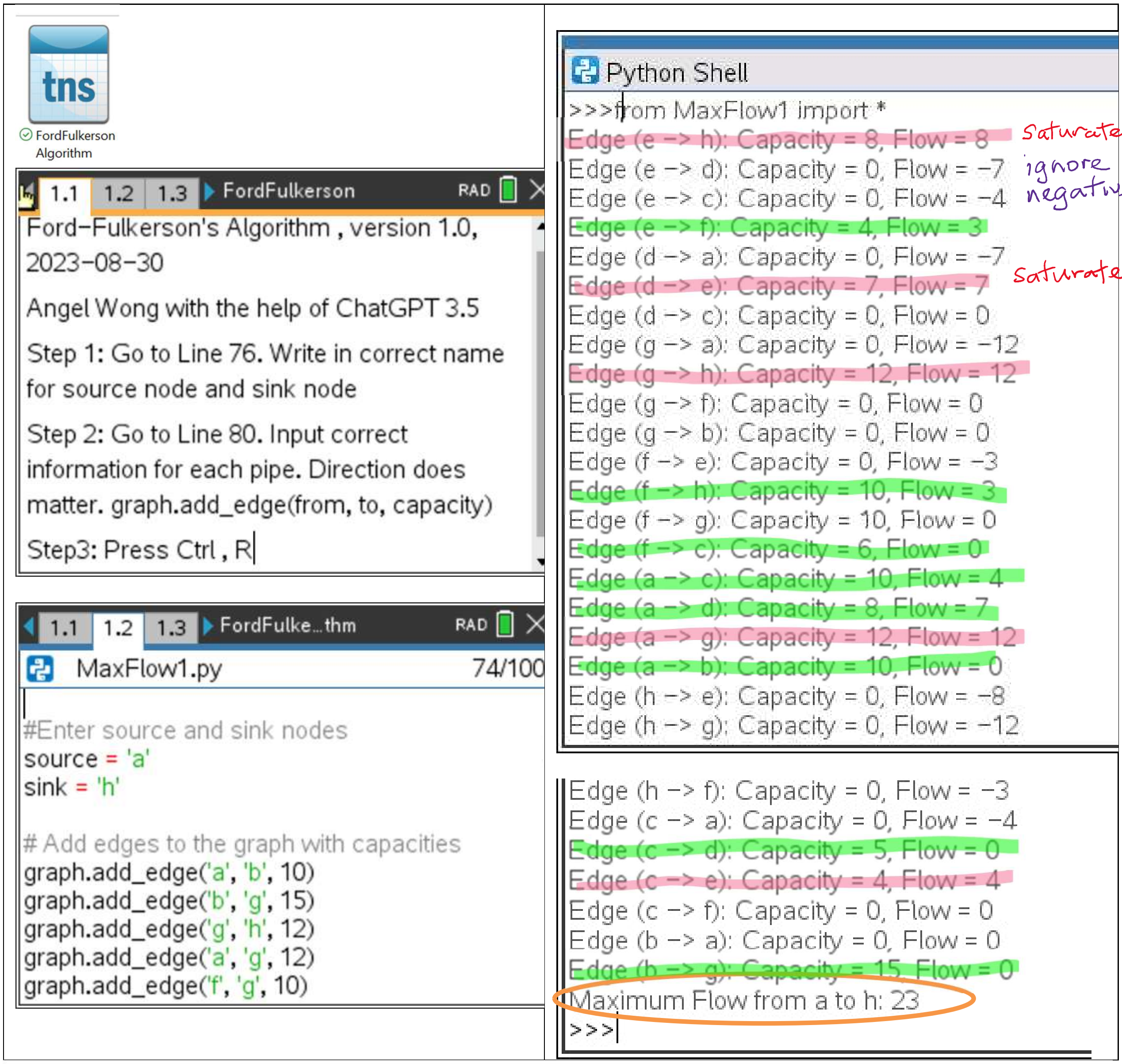
Path from entrance A (source) to exit H (sink)? A – B – G – H 10 15 12 min 10 children / min AB is now **saturated**		

A – G – H 12 2 min 2 children/min GH is now **saturated**		

A – C – E – F – H 10 4 4 10 min 4 children / min CE, EF are now **saturated**	
A – C – E – F – G – H 6 0 0 10 0 min 0 child/min	
A – D – E – H 8 7 8 min 7 child/min With the highlighted red saturated pipes, we can't push any more water (children) through the network. Hence, sum all min number on each path 10 + 2 + 4 + 7 = 23 children/min (Max flow)	

Notice, with different path checking order, it may result in a different optimal flow diagram.

Method 3 – Ford-Fulkerson's algorithm on CAS (Python program)



Ford–Fulkerson's Algorithm , version 1.0, 2023–08–30

Angel Wong with the help of ChatGPT 3.5

Step 1: Go to Line 76. Write in correct name for source node and sink node

Step 2: Go to Line 80. Input correct information for each pipe. Direction does matter. graph.add_edge(from, to, capacity)

Step3: Press Ctrl , R

```
#Enter source and sink nodes
source = 'a'
sink = 'h'

# Add edges to the graph with capacities
graph.add_edge('a', 'b', 10)
graph.add_edge('b', 'g', 15)
graph.add_edge('g', 'h', 12)
graph.add_edge('a', 'g', 12)
graph.add_edge('f', 'g', 10)
```

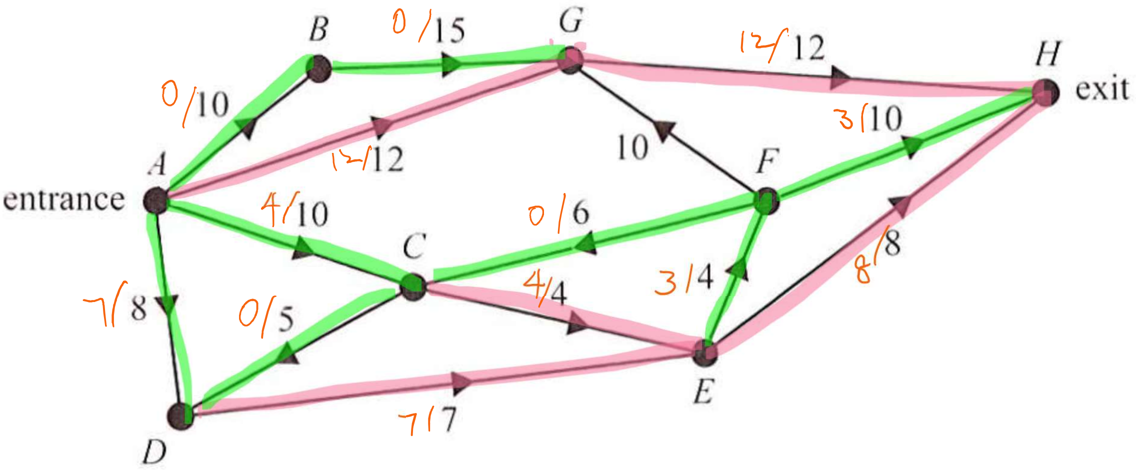
```
>>>from MaxFlow1 import *
Edge (e -> h): Capacity = 8, Flow = 8
Edge (e -> d): Capacity = 0, Flow = -7
Edge (e -> c): Capacity = 0, Flow = -4
Edge (e -> f): Capacity = 4, Flow = 3
Edge (d -> a): Capacity = 0, Flow = -7
Edge (d -> e): Capacity = 7, Flow = 7
Edge (d -> c): Capacity = 0, Flow = 0
Edge (g -> a): Capacity = 0, Flow = -12
Edge (g -> h): Capacity = 12, Flow = 12
Edge (g -> f): Capacity = 0, Flow = 0
Edge (g -> b): Capacity = 0, Flow = 0
Edge (f -> e): Capacity = 0, Flow = -3
Edge (f -> h): Capacity = 10, Flow = 3
Edge (f -> g): Capacity = 10, Flow = 0
Edge (f -> c): Capacity = 6, Flow = 0
Edge (a -> c): Capacity = 10, Flow = 4
Edge (a -> d): Capacity = 8, Flow = 7
Edge (a -> g): Capacity = 12, Flow = 12
Edge (a -> b): Capacity = 10, Flow = 0
Edge (h -> e): Capacity = 0, Flow = -8
Edge (h -> g): Capacity = 0, Flow = -12
```

```
Edge (h -> f): Capacity = 0, Flow = -3
Edge (c -> a): Capacity = 0, Flow = -4
Edge (c -> d): Capacity = 5, Flow = 0
Edge (c -> e): Capacity = 4, Flow = 4
Edge (c -> f): Capacity = 0, Flow = 0
Edge (b -> a): Capacity = 0, Flow = 0
Edge (b -> g): Capacity = 15, Flow = 0
Maximum Flow from a to h: 23
>>>
```

saturated

ignore negative flow

saturated



Notice, with different path checking order, it may result in a different optimal flow diagram!!

Question 40

One path in the maze is to be changed. Which one of these five changes would lead to the largest increase in flow from entrance to exit?

A. Increasing the capacity of flow along the edge *CE* to 12
B. Increasing the capacity of flow along the edge *FH* to 14
C. Increasing the capacity of flow along the edge *GH* to 16
D. Reversing the direction of flow along the edge *CF*
E. Reversing the direction of flow along the edge *GF*

From Flow analysis done previously
Continue from Method 2 of Question 39 above

Option A: Increase *CE* to 12

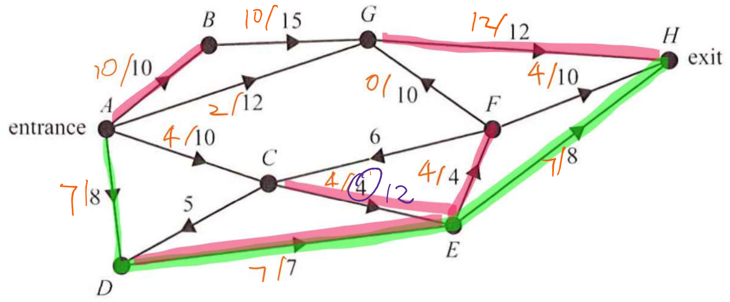
This means extra (12-4)=8 units can flow through *CE*.

However, paths go from source to sink via *CE*:

A – C – E – F – H
A – C – E – F – C – E – H but EF and CE are saturated hence can't push any more through

A – C – E – H
6 8 1 can push one unit more

Hence, Max flow increase from 23 to (23+1) = 24 children / min



Option B: Increase *FH* to 14

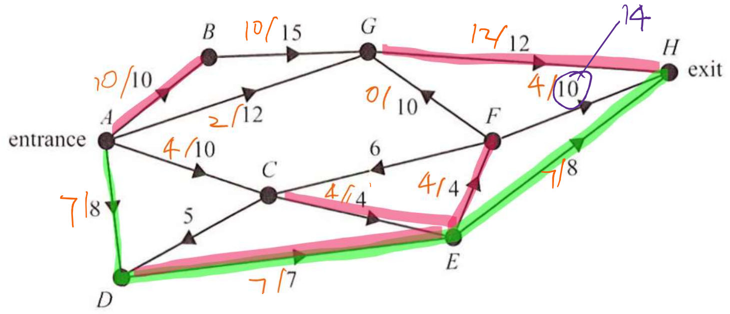
This means extra (14-4)=10 units can flow through *FH*.

However, two paths go from source to sink via *FH*:

A – C – E – F – H but EF is saturated hence can't push any more through

A – D – E – F – H but DE, EF are saturated hence can't push any more through

Hence, Max flow stays at 23 children/min



Option C: Increase *GH* to 16

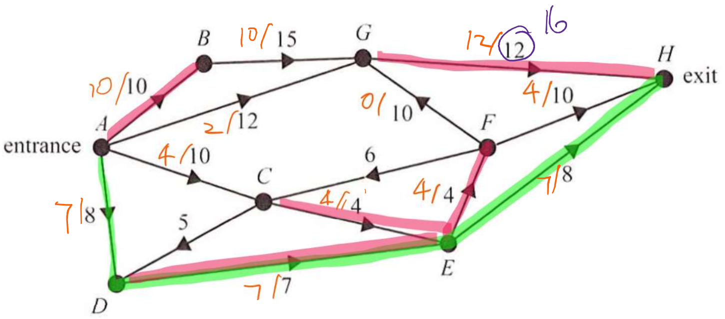
This means extra (16-12)=4 units can flow through *GH*.
However, three paths go from source to sink via *GH*:

A – B – G – H but AB is saturated hence can't push any more through

A – G – H
10 4 Hence can push through 4 units more

A – D – E – F – G – H but DE, EF are saturated hence can't push any more through

Hence, Max flow becomes (23+4)=27 children/min

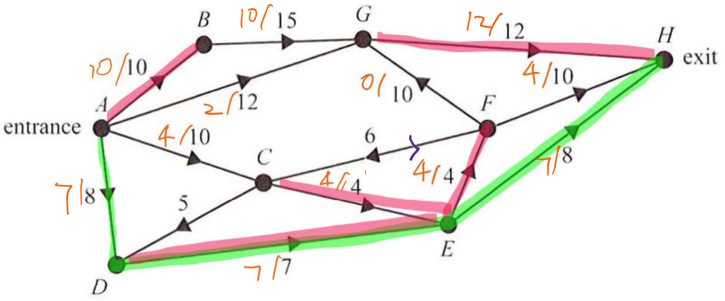


Option D: Reverse *CF*

This means extra 6 units can flow through *CF*.
However, path go from source to sink via *CF*:

A – C – F – H
6 6 6 Hence push through 6 units more

Hence, Max flow becomes (23+6)=29 children/min

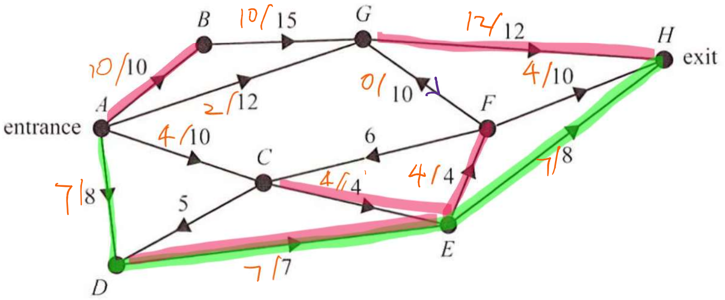


Option E: Reverse *GF*

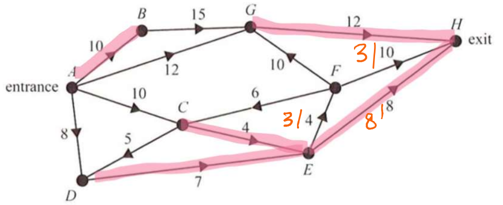
This means extra 6 units can flow through *GF*.
However, the paths go from source to sink via *GF*:

A – B – G – F – H
A – G – F – C – E - H
A – G – F – C – E – F – H
A – G – F – C - D – E - H
A – G – F – C – D – E – F – H
These three paths all go through saturated pipes so no increase in flow

A – G – F – H
10
10 6 Hence push through 6 units more



Does this mean Max flow becomes (23+6)=29 children/min ??
Remember optimal flow diagram has more than one solution. Another way could be making EH saturated rather than EF and redistributed flow of 4 in EF to 1 in EH and 3 in FH



Then A – G – F – H
10 10 7 Hence extra 7 units increase

Hence, Max flow becomes (23+7)=30 children/min

However, if we use Ford-Fulkerson's Algorithm on CAS (Python Program), it is just a matter of making relevant change for each option and check which option gives the Maximum Flow.

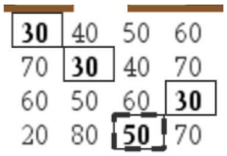
Session C – Hungarian Algorithm

Allocate to minimize (Cost, Time etc…)

Four people, Abe, Bailey, Chris and Donna, are each to be allocated one of four tasks. Each person can complete each of the four tasks in a set time. These times, in minutes, are shown in the table below.

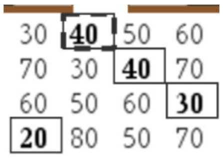
	Abe	Bailey	Chris	Donna
Task W	30	40	50	60
Task X	70	30	40	70
Task Y	60	50	60	30
Task Z	20	80	50	70

Try out some possible combinations:



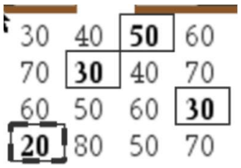
30	40	50	60
70	**30**	40	70
60	50	60	**30**
20	80	**50**	70

Sum
= 30 + 30 + 30 + 50 =
140min



30	**40**	50	60
70	30	**40**	70
60	50	60	**30**
20	80	50	70

Sum
= 40 + 40 + 30 + 20 = 130
min



30	40	**50**	60
70	**30**	40	70
60	50	60	**30**
20	80	50	70

Sum
= 50 + 30 + 30 + 20 = 130
min

Apply the **Hungarian Algorithm** to allocate each person a different task so the total time for these four people to complete these four tasks is minimized.

Aim: To find the optimal allocation of task to each person. **One zero per row or column**

Step One – Row Reduction by smallest in row

30	40	50	60	**-30**
70	30	40	70	**-30**
60	50	60	30	**-30**
20	80	50	70	**-20**

0	10	20	30
40	0	10	40
30	20	30	0
0	60	30	50

↑

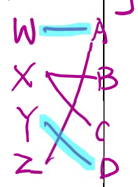
No ‘0’ in this Column!

Step Two – Column Reduction

0	10	20	30
40	0	10	40
30	20	30	0
0	60	30	50
-0	**-0**	**-10**	**-0**

	A	B	C	D
W	0	10	10	30
X	40	0	0	40
Y	30	20	20	0
Z	0	60	20	50

Try allocating



Cover multiple zeros with one line. Use minimum lines to cover all zeros possible.

One way of covering ‘0’s with straight line.

Further adjustment

0	10	10	30
40	0	0	40
30	20	20	0
0	60	20	50

- Add smallest uncovered element to elements covered by two lines.
- Subtract smallest uncovered element from all uncovered elements

0	0	0	30
50	0	0	50
30	10	10	0
0	50	10	50

Another way of covering ‘0’s with straight line

Further adjustment

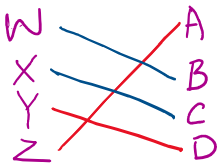
0	10	10	30
40	0	0	40
30	20	20	0
0	60	20	50

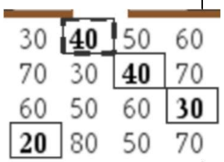
- Add smallest uncovered element to elements covered by two lines.
- Subtract smallest uncovered element from all uncovered elements

0	0	0	20
50	0	0	40
40	20	20	0
0	50	10	40

Start to Allocate

Start with row and/column with 1 ‘0’
This allocates Y to D and Z to A

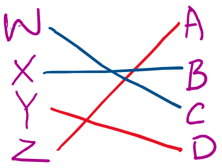


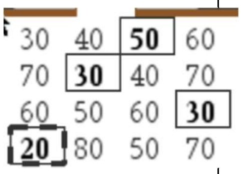


30	**40**	50	60
70	30	**40**	70
60	50	60	**30**
20	80	50	70

Allocate W to B and X to C and check time:
Min time = 40+40+30+20=130 min

Try also
Allocate W to C and X to B and check time:





30	40	**50**	60
70	**30**	40	70
60	50	60	**30**
20	80	50	70

Min time = 50+30+30+20=130 min

[2022 NHT Further Maths paper 1]

Question 6

Five builders, Amida, Boris, Clea, Drew and Enzo, are working on a construction.

The construction has five components that must all be completed.

Each builder will be allocated only one component.

The table below shows the time, in hours, it would take each builder to complete each component of the construction.

	Amida	**Boris**	**Clea**	**Drew**	**Enzo**
1st component	6	6	5	7	7
2nd component	5	7	6	7	4
3rd component	8	5	7	5	9
4th component	7	7	8	6	6
5th component	4	4	4	5	5

The components of this construction must be completed in numerical order: 1st, 2nd, 3rd, 4th and 5th.

Each builder will be assigned to one component to ensure that construction is completed in the minimum time possible.

Which one of the following statements is **not** true?

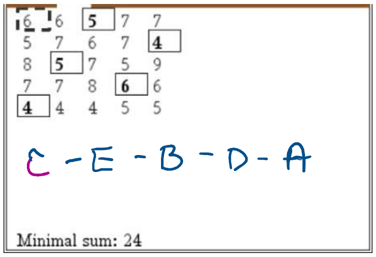
[✓ **A.**] Amida will complete her allocated component before Boris.

B. Boris will complete his allocated component after Clea.

C. Clea will complete her allocated component before Drew.

D. Drew will complete his allocated component after Enzo.

E. Enzo will complete his allocated component before Amida.



6	6	**5**	7	7
5	7	6	7	**4**
8	**5**	7	5	9
7	7	8	**6**	6
4	4	4	5	5

C - E - B - D - A

Minimal sum: 24

[2022 VCAA Further Maths paper 1]

An athletics club needs to select one team of four athletes.

The team is required to have one long jump, one high jump, one shot put and one javelin competitor.

The following table shows the best distances, in metres, for each athlete for each event.

Athlete	Long jump (m)	High jump (m)	Shot put (m)	Javelin (m)
Eve	4.8	1.7	13.1	40.9
Harsha	4.8	1.6	13.9	39.5
Shona	5.1	1.8	14.4	41.2
Taylor	4.8	1.7	12.8	39.8

The athletics club will allocate each athlete to one event in order to maximise the total distance that the team jumps and throws.

Which allocation of athlete to event must occur in order to maximise the total distance?

A.

long jump	high jump	shot put	javelin
Shona	Harsha	Eve	Taylor

B.

long jump	high jump	shot put	javelin
Shona	Taylor	Harsha	Eve

C.

long jump	high jump	shot put	javelin
Eve	Harsha	Taylor	Shona

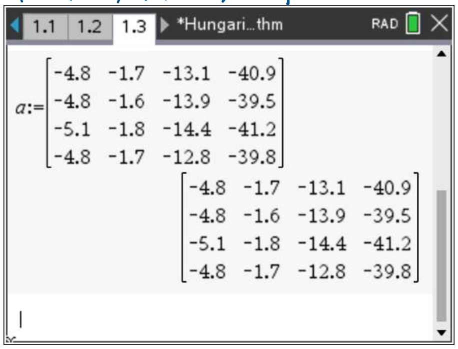
[✓ **D.**]

long jump	high jump	shot put	javelin
Harsha	Taylor	Shona	Eve

E.

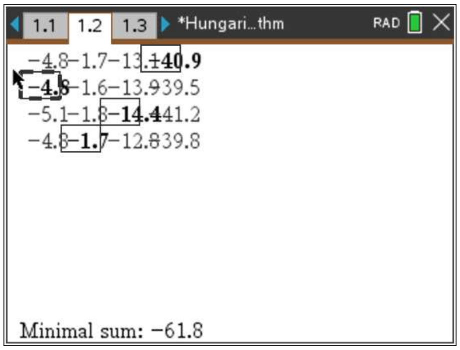
long jump	high jump	shot put	javelin
Harsha	Taylor	Eve	Shona

To maximize, define with 'negative'



$$a:=\begin{bmatrix}-4.8 & -1.7 & -13.1 & -40.9\\ -4.8 & -1.6 & -13.9 & -39.5\\ -5.1 & -1.8 & -14.4 & -41.2\\ -4.8 & -1.7 & -12.8 & -39.8\end{bmatrix}$$

$$\begin{bmatrix}-4.8 & -1.7 & -13.1 & -40.9\\ -4.8 & -1.6 & -13.9 & -39.5\\ -5.1 & -1.8 & -14.4 & -41.2\\ -4.8 & -1.7 & -12.8 & -39.8\end{bmatrix}$$



```
−4.8−1.7−13.140.9
−4.8−1.6−13.939.5
−5.1−1.8−14.441.2
−4.8−1.7−12.839.8
```

Minimal sum: −61.8

[2024 NHT General Maths Paper 1 Q 36]

Question 36

Four students, Peggy, Quincy, Radley and Sarah, are grouped together to complete a project. The project is in four parts, labelled W, X, Y and Z. Each student must complete one part of the project.

The table below shows each student's estimate of the score they will receive if they complete each section.

	Peggy	Quincy	Radley	Sarah
W	12	19	18	16
X	16	15	15	16
Y	10	16	17	15
Z	19	20	18	18

Based on the estimates, which allocation of project parts will maximise the students' group score on the project?

A.

W	Quincy
X	Sarah
Y	Radley
Z	Peggy

B.

W	Radley
X	Peggy
Y	Quincy
Z	Sarah

C.

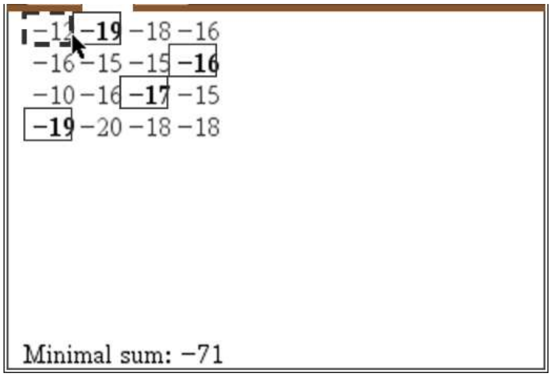
W	Sarah
X	Quincy
Y	Peggy
Z	Radley

D.

W	Radley
X	Peggy
Y	Sarah
Z	Quincy

E.

W	Sarah
X	Peggy
Y	Radley
Z	Quincy



```
−12 −19 −18 −16
−16 −15 −15 −16
−10 −16 −17 −15
−19 −20 −18 −18

Minimal sum: −71
```

Ans: A

[2024 VCAA General Maths Paper 1 Q 39]

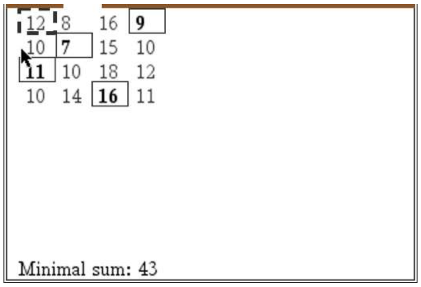
2024 VCE General Mathematics Examination 1 Page 25 of 28

Question 39

Anush, Blake, Carly and Dexter are workers on a construction site. They are each allocated one task.

The time, in hours, it takes for each worker to complete each task is shown in the table below.

	Task 1	Task 2	Task 3	Task 4
Anush	12	8	16	9
Blake	10	7	15	10
Carly	11	10	18	12
Dexter	10	14	16	11



12 8 16 **9**
10 **7** 15 10
11 10 18 12
10 14 **16** 11

Minimal sum: 43

The tasks must be completed sequentially and in numerical order: Task 1, Task 2, Task 3 and then Task 4.

Management makes an initial allocation of tasks to minimise the amount of time required, but then decides that it takes the workers too long.

Another worker, Edgar, is brought in to complete one of the tasks.

His completion times, in hours, are listed below.

	Task 1	Task 2	Task 3	Task 4
Edgar	9	5	14	8

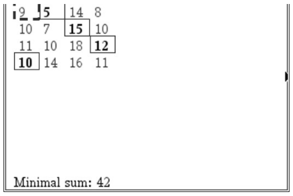
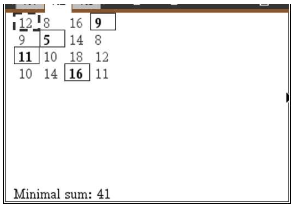
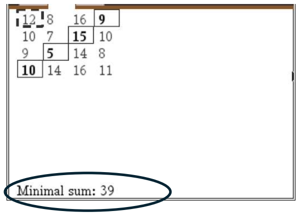
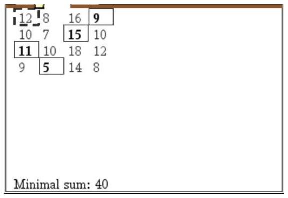
When a new allocation is made and Edgar takes over one of the tasks, the minimum total completion time compared to the initial allocation will be reduced by

A. 1 hour.

B. 2 hours.

C. 3 hours.

D. 4 hours.

Edgar replaces Anush	Edgar replaces Blake	Edgar replaces Carly	Edgar replaces Dexter
 9 **5** 14 8 10 7 **15** 10 11 10 18 **12** **10** 14 16 11 Minimal sum: 42	 12 8 16 **9** 9 **5** 14 8 **11** 10 18 12 10 14 **16** 11 Minimal sum: 41	 12 8 16 **9** 10 7 **15** 10 9 **5** 14 8 **10** 14 16 11 Minimal sum: 39 **Ans = 43 – 39 = 4 hr**	 12 8 16 **9** 10 7 **15** 10 **11** 10 18 12 9 **5** 14 8 Minimal sum: 40

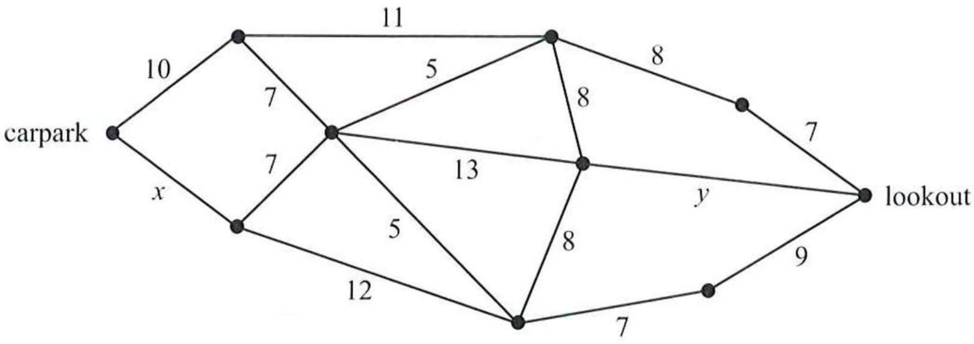
Session D – Dijkstra’s Algorithm

[2024 VCE General Maths Paper 1]

Question 37

The network below represents paths through a park from the carpark to a lookout.

The vertices represent various attractions, and the numbers on the edges represent the distances between them in metres.



The shortest path from the carpark to the lookout is 34 m.

This can be achieved when

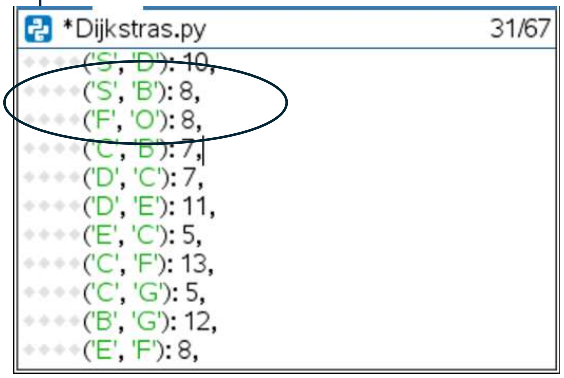
A. $x = 8$ and $y = 8$

B. $x = 9$ and $y = 7$

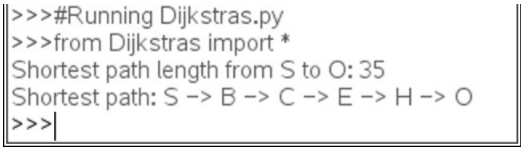
C. $x = 10$ and $y = 6$

D. $x = 11$ and $y = 5$

Option A

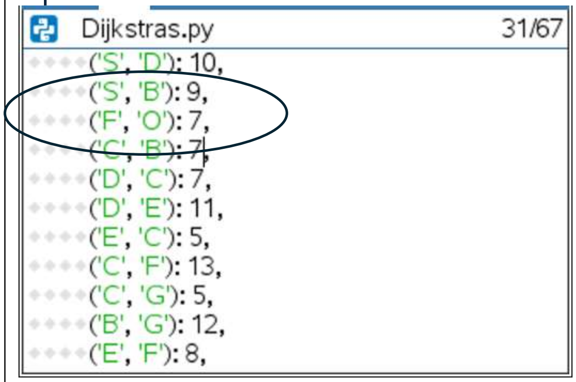


```
    ('S', 'D'): 10,
    ('S', 'B'): 8,
    ('F', 'O'): 8,
    ('C', 'B'): 7,
    ('D', 'C'): 7,
    ('D', 'E'): 11,
    ('E', 'C'): 5,
    ('C', 'F'): 13,
    ('C', 'G'): 5,
    ('B', 'G'): 12,
    ('E', 'F'): 8,
```

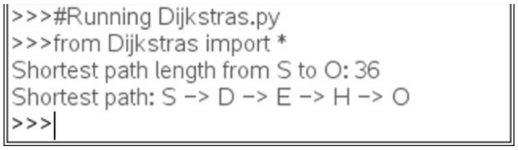


```
>>>#Running Dijkstras.py
>>>from Dijkstras import *
Shortest path length from S to O: 35
Shortest path: S -> B -> C -> E -> H -> O
>>>
```

Option B

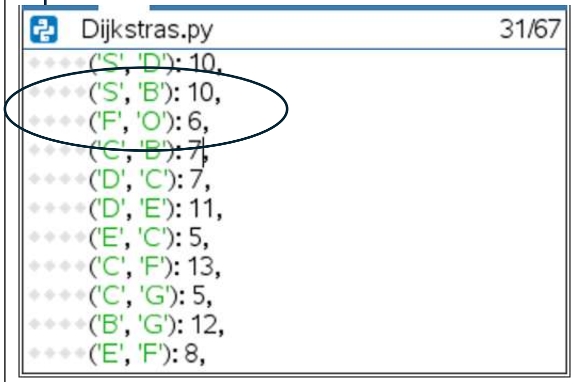


```
    ('S', 'D'): 10,
    ('S', 'B'): 9,
    ('F', 'O'): 7,
    ('C', 'B'): 7,
    ('D', 'C'): 7,
    ('D', 'E'): 11,
    ('E', 'C'): 5,
    ('C', 'F'): 13,
    ('C', 'G'): 5,
    ('B', 'G'): 12,
    ('E', 'F'): 8,
```

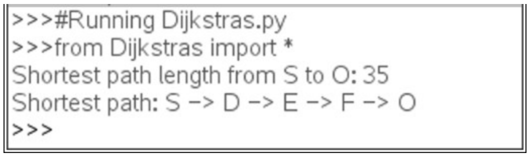


```
>>>#Running Dijkstras.py
>>>from Dijkstras import *
Shortest path length from S to O: 36
Shortest path: S -> D -> E -> H -> O
>>>
```

Option C

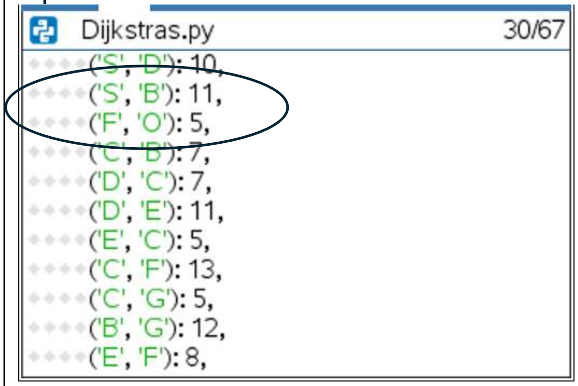


```
    ('S', 'D'): 10,
    ('S', 'B'): 10,
    ('F', 'O'): 6,
    ('C', 'B'): 7,
    ('D', 'C'): 7,
    ('D', 'E'): 11,
    ('E', 'C'): 5,
    ('C', 'F'): 13,
    ('C', 'G'): 5,
    ('B', 'G'): 12,
    ('E', 'F'): 8,
```

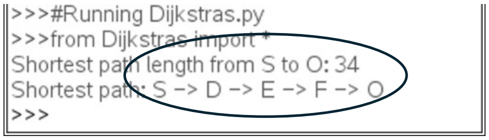


```
>>>#Running Dijkstras.py
>>>from Dijkstras import *
Shortest path length from S to O: 35
Shortest path: S -> D -> E -> F -> O
>>>
```

Option D



```
    ('S', 'D'): 10,
    ('S', 'B'): 11,
    ('F', 'O'): 5,
    ('C', 'B'): 7,
    ('D', 'C'): 7,
    ('D', 'E'): 11,
    ('E', 'C'): 5,
    ('C', 'F'): 13,
    ('C', 'G'): 5,
    ('B', 'G'): 12,
    ('E', 'F'): 8,
```



```
>>>#Running Dijkstras.py
>>>from Dijkstras import *
Shortest path length from S to O: 34
Shortest path: S -> D -> E -> F -> O
>>>
```

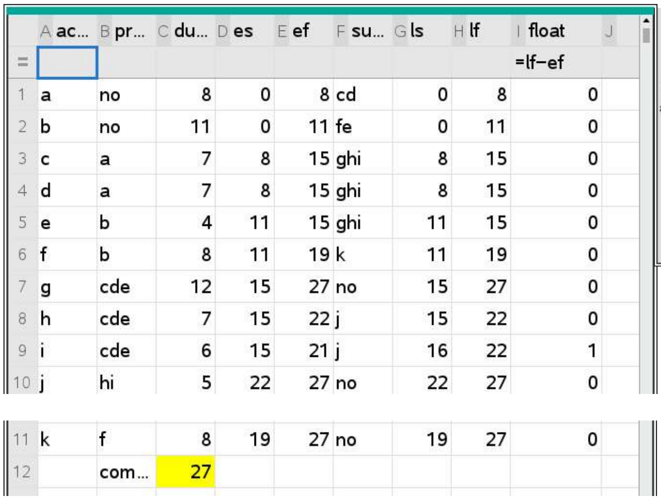
Ans: D

Appendix

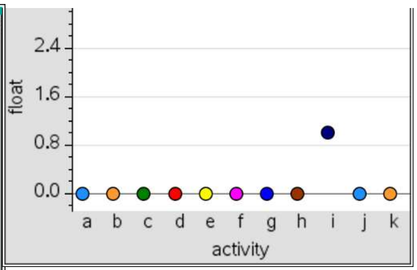
1	A
2	B
3	C
4	D
5	E
6	F
7	G
8	H
9	I
10	J
11	K
12	L
13	M

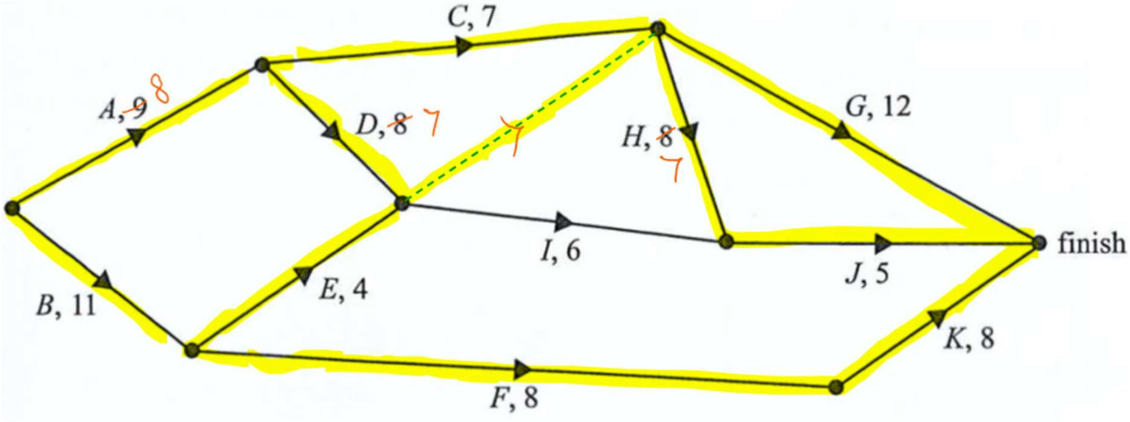
14	N
15	O
16	P
17	Q
18	R
19	S
20	T
21	U
22	V
23	W
24	X
25	Y
26	Z

Reduce A by 1 week:

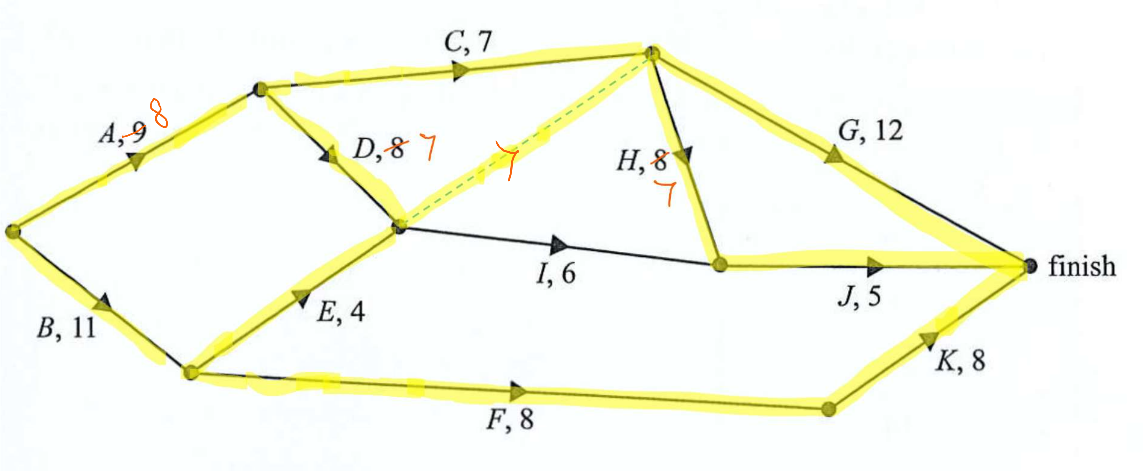


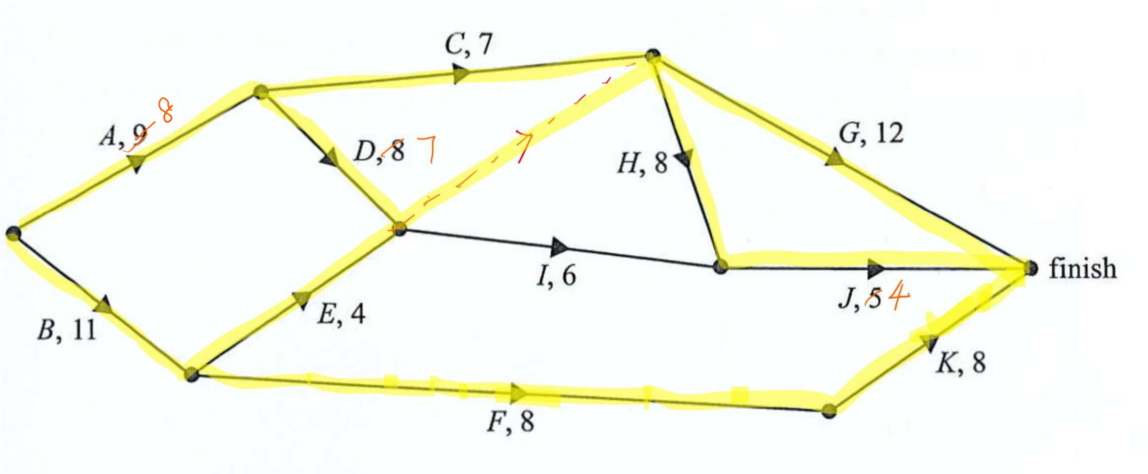
	A ac…	B pr…	C du…	D es	E ef	F su…	G ls	H lf	I float	J
=									=lf−ef	
1	a	no	8	0	8	cd	0	8	0	
2	b	no	11	0	11	fe	0	11	0	
3	c	a	7	8	15	ghi	8	15	0	
4	d	a	7	8	15	ghi	8	15	0	
5	e	b	4	11	15	ghi	11	15	0	
6	f	b	8	11	19	k	11	19	0	
7	g	cde	12	15	27	no	15	27	0	
8	h	cde	7	15	22	j	15	22	0	
9	i	cde	6	15	21	j	16	22	1	
10	j	hi	5	22	27	no	22	27	0	
11	k	f	8	19	27	no	19	27	0	
12		com…	27							

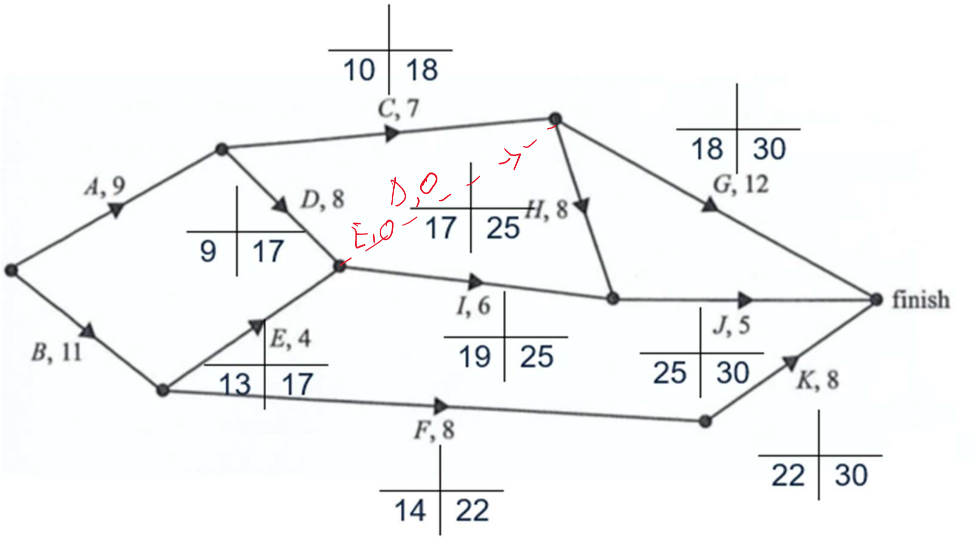




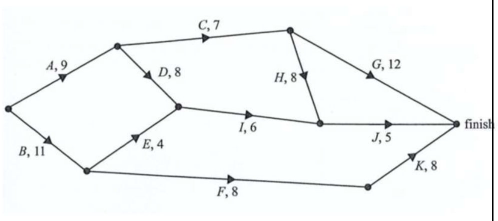
1. A (8) - $10 000, D (7) - $10 000, H (7) - $10 000, total $30 000 so far
2. Can't reduce C, D, H further as min weeks of each activity is 7
3. A (8) + D (7) = 15 is the same as B (11) + E (4). Reduce both A and B by 1 week

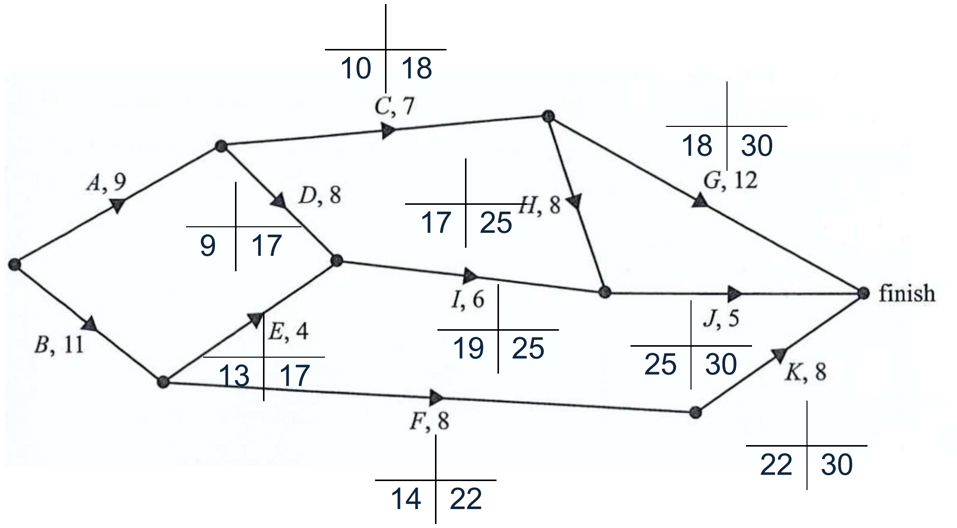


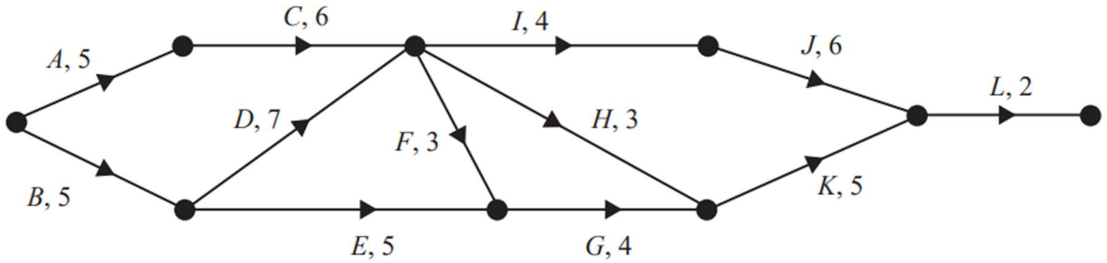

C, 7
A, ~~9~~ 8
D, ~~8~~ 7
H, 8
G, 12
I, 6
J, ~~5~~ 4
B, 11
E, 4
K, 8
F, 8
finish

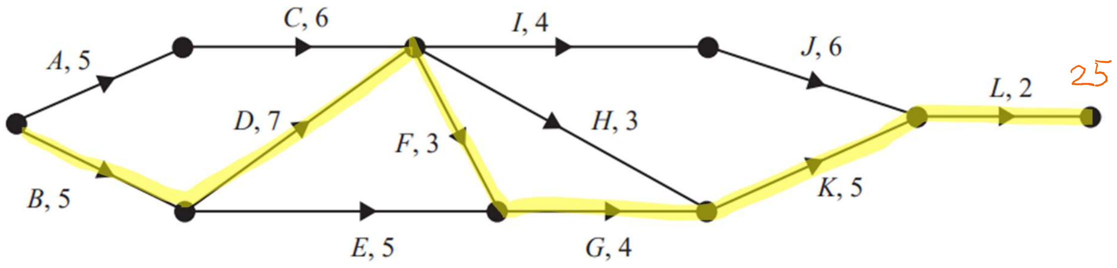

10 | 18
C, 7
18 | 30
G, 12
A, 9
D, 8
D, 0
E, 0
9 | 17
17 | 25
H, 8
I, 6
19 | 25
J, 5
25 | 30
E, 4
13 | 17
B, 11
K, 8
F, 8
14 | 22
22 | 30
finish

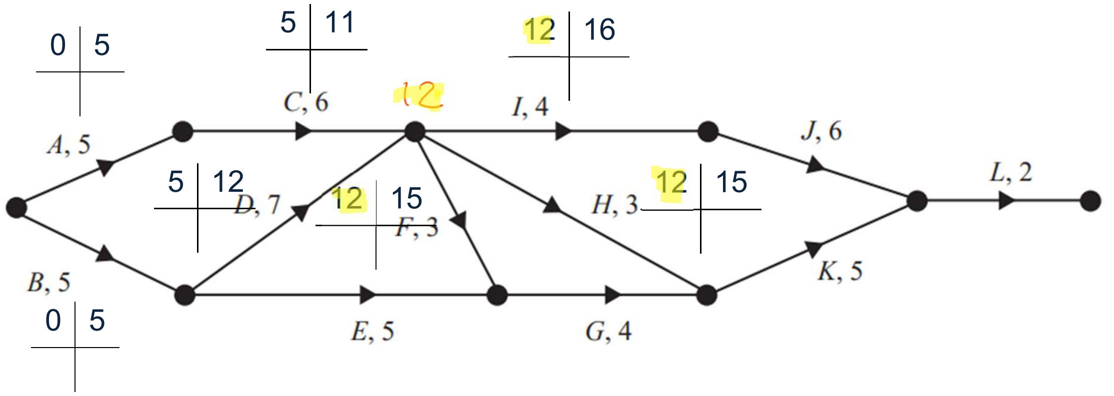
Activity	New LST (weeks)	Old LST	New LFT = LST + Duration	Old LFT
A	0	0	0+9=9	9
B	2	2	2+11=13	13
C	10	9	10+7=18	16
D	9	10	9+8=17	18
E	13	14	13+4=17	18
F	14	13	14+8=22	21
G	18	17	18+12=30	29
H	17	16	17+8=25	24
I	19	18	19+6=25	24
J	25	24	25+5=30	29
K	22	21	22+8=30	29

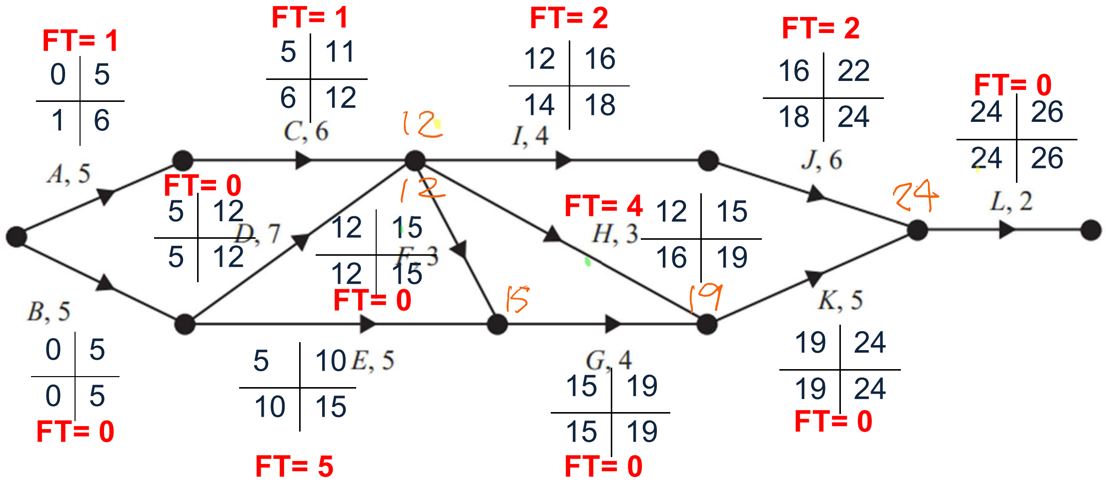


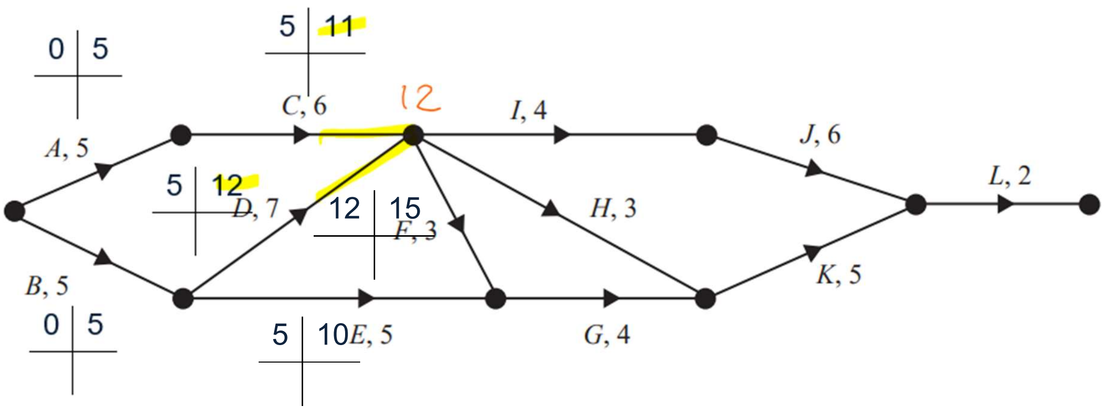



A, 5
B, 5
C, 6
D, 7
E, 5
F, 3
G, 4
H, 3
I, 4
J, 6
K, 5
L, 2


A, 5
B, 5
C, 6
D, 7
E, 5
F, 3
G, 4
H, 3
I, 4
J, 6
K, 5
L, 2
25


0 | 5
5 | 11
12 | 16
12
5 | 12
12 | 15
12 | 15
0 | 5
A, 5
B, 5
C, 6
D, 7
E, 5
F, 3
G, 4
H, 3
I, 4
J, 6
K, 5
L, 2


FT= 1
0 | 5
1 | 6
FT= 1
5 | 11
6 | 12
FT= 2
12 | 16
14 | 18
FT= 2
16 | 22
18 | 24
FT= 0
24 | 26
24 | 26
12
12
FT= 0
5 | 12
5 | 12
12 | 15
12 | 15
FT= 0
FT= 4
12 | 15
16 | 19
15
19
24
0 | 5
0 | 5
FT= 0
5 | 10
10 | 15
FT= 5
15 | 19
15 | 19
FT= 0
19 | 24
19 | 24
FT= 0
A, 5
B, 5
C, 6
D, 7
E, 5
F, 3
G, 4
H, 3
I, 4
J, 6
K, 5
L, 2



0 | 5
A, 5
5 | 11
C, 6
12
I, 4
J, 6
L, 2
5 | 12
D, 7
12 | 15
F, 3
H, 3
K, 5
B, 5
0 | 5
5 | 10
E, 5
G, 4

List all routes through the network:

1.	A (5)	C (6)	I (4)	J (6)	L (2)		5+6+4+6+2 = 23
2.	A (5)	C (6)	H (3)	K (5)	L (2)		5+6+3+5+2= 21
3.	A (5)	C (6)	F (3)	G (4)	K (5)	L (2)	5+6+3+4+5+2 = 25
4.	B (5)	D (7)	I (4)	J (6)	L (2)		5+7+4+6+2 = 24
5.	B (5)	E (5)	G (4)	K (5)	L (2)		5+5+4+5+2 = 21
6.	B (5)	D (7)	F (3)	G (4)	K (5)	L (2)	5+7+3+4+5+2= 26
7.	B (5)	D (7)	H (3)	K (5)	L (2)		5+7+3+5+2 = 22

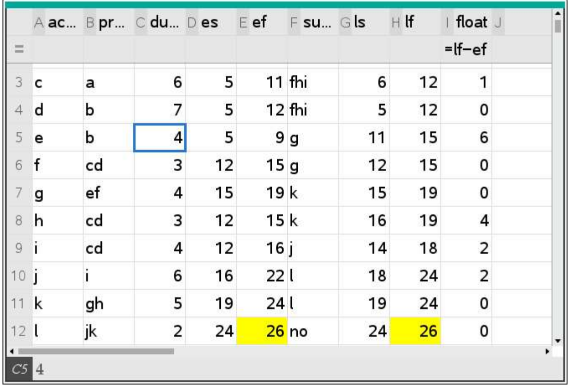
Additional Notes: Notice the longest path (26 days) is the minimum completion time.

Say if we choose the cheapest activity to crash. In this case : E - $1000

1.	A (5)	C (6)	I (4)	J (6)	L (2)		5+6+4+6+2 = 23
2.	A (5)	C (6)	H (3)	K (5)	L (2)		5+6+3+5+2= 21
3.	A (5)	C (6)	F (3)	G (4)	K (5)	L (2)	5+6+3+4+5+2 = 25
4.	B (5)	D (7)	I (4)	J (6)	L (2)		5+7+4+6+2 = 24
5.	B (5)	E (5)	G (4)	K (5)	L (2)		5+5+4+5+2 = 21
6.	B (5)	D (7)	F (3)	G (4)	K (5)	L (2)	5+7+3+4+5+2= 26
7.	B (5)	D (7)	H (3)	K (5)	L (2)		5+7+3+5+2 = 22

Crashing E by 1 or 2 days is only going to reduce path 5 from 21 to 20 or 19 but it is not going to change the completion time of 26 days.

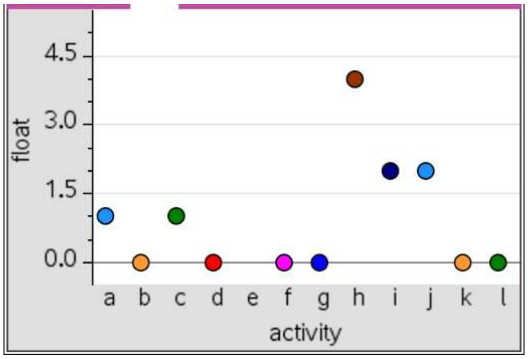
Check on CAS if Spreadsheet is set
Change duration of activity E to 4 and then to 3 and see that completion time is still 26 days.



	A ac...	B pr...	C du...	D es	E ef	F su...	G ls	H lf	I float	J
=									=lf−ef	
3	c	a	6	5	11	fhi	6	12	1	
4	d	b	7	5	12	fhi	5	12	0	
5	e	b	4	5	9	g	11	15	6	
6	f	cd	3	12	15	g	12	15	0	
7	g	ef	4	15	19	k	15	19	0	
8	h	cd	3	12	15	k	16	19	4	
9	i	cd	4	12	16	j	14	18	2	
10	j	i	6	16	22	l	18	24	2	
11	k	gh	5	19	24	l	19	24	0	
12	l	jk	2	24	26	no	24	26	0	

C5 4

Check Critical Path:
B D F G K L



e.